ASTRO SCIENCES CENTER

Report No. 1. 4 T1 No. 3

INTERIM SUMMARY OF CONIC SECTION
TRAJECTORY SYSTEM, INCLUDING EARTH-
JUPITER TRAJECTORIES, 1966-1978

Lunar and Planetary Programs Office
National Aeronautics and Space Administration

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INTERIM SUMMARY OF CONIC SECTION TRAJECTORY SYSTEM
INCLUDING EARTH-JUPITER TRAJECTORIES, 1966-1978

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Chicago, Illinois

for

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ABSTRACT

INTERIM SUMMARY OF CONIC SECTION TRAJECTORY CALCULATING SYSTEM

A computerized system has been devised for performing impulsive trajectory calculations on an IBM 7090 computer from any object in the Astro Sciences Center catalogue of solar system targets to any other object. All objects are assumed to be in conic section orbits; thrusting is impulsive; one gravitating body is considered at a time. The system is divided into subroutines for flexibility. It is being used for determining velocity increment requirements and other characteristics of various missions. The system is now working: a limited number of additions and improvements are planned. The ephemeris tape now contains orbital data for the Sun and planets, 87 periodic comets and 1563 numbered asteroids. Extensive use has been made of techniques and data from JPL in programming this system. Thirty two curves related to flights from Earth to Jupiter in 1966 through 1978 illustrate the capabilities of the system.

APPROVED:

L. Reiffel, Director
Astro Sciences Center

INTERIM SUMMARY OF CONIC SECTION TRAJECTORY SYSTEM, INCLUDING EARTH-JUPITER TRAJECTORIES, 1966-1978

I. GENERAL SYSTEM DESCRIPTION

This system is designed to provide, in a fully automatic manner, impulsive trajectory calculations from any object in our catalogue of solar system targets to any other object; the system also provides the capability of using its parts for other calculations. The standard conic section trajectory equations are evaluated on the IBM 7090 computer in FORTRAN II language; simplicity, flexibility and ease of use were the guiding factors in devising the system. All bodies are assumed to be in elliptical, hyperbolic or parabolic orbits.

The main trajectory calculation proceeds as follows:

Given time of launch from one body and time of arrival at a second, the computer calculates V_{SE}, the heliocentric spacecraft orbital velocity minus the earth's* heliocentric orbital velocity at launch time (i.e. the asymptotic escape velocity of the spacecraft from a geocentric orbit), and V_{ST} the spacecraft to target velocity difference at arrival time.

In addition the code calculates:

$\theta_{SE} = V_S \cdot V_E$	the angle between the velocity of the spacecraft at launch and the velocity of the earth at time of launch.
$\theta_{ST} = V_S \cdot V_T$	the angle between the velocity of the spacecraft at arrival at the target and the velocity of the target at time of arrival.
θ_{SPE}	the angle between the velocity of the spacecraft at launch and the plane of the earth's orbit.

* Earth is used as the standard launch base in this report. However the code is general and will handle flights from any object to any other object on the tape.

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Θ_{SPT}	the angle between the velocity of the space-craft at arrival at the target and the plane of the target's orbit.
ΔR_{ET}	the (communications) distance between the earth and target, at arrival of the spacecraft at the target.
$\Delta R_{S-SN, MIN}$	the minimum spacecraft to sun distance during the flight.
Θ_{E-SN-T}	the earth-sun-target angle, at time of space-craft arrival at the target.
Θ_{LCHLIM}	the range of azimuth's at Cape Canaveral from which a direct launch into the plane of the transfer orbit cannot be made.

An optional output is the positions of earth, target, spacecraft and any other three objects throughout the flight. All standard output may be plotted, as an option, on the IBM 1401 printer.

An additional code has been prepared to order the comet and asteroid data on the ephemeris tape: ordering is by semi-major axis, eccentricity, inclination, period, perihelion distance or time or perihelion.

A separate code has been written for calculating the velocity increment necessary to put an incoming spacecraft, approaching a planet hyperbolically, into an elliptic orbit around the planet.

Calculation time, without IBM 1401 plotting or other options, is about 4 trajectories per second of IBM 7090 time for the main code.

II.   MAJOR SYSTEM SUBSECTIONS

BUMP:
: BUMP is the main supervisory program which controls input, output and the flow of computations, and performs some of the simpler calculations and scaling for output.

FDPLNE:
: FDPLNE is a subroutine which finds the plane in which two radius vectors lie.

LMCON:
: LMCON is a subroutine which finds the parameters of the spacecraft orbit, given the plane from FDPLNE, the earth and target positions and the time of flight LMCON is based largely on a JPL memo by V. C. Clark, Jr.

KEPLER:
: KEPLER is a subroutine which solves Kepler's transcedental equation for elliptical or hyperbolic trajectories.

TIME:
: TIME is a subroutine which finds the time in seconds after 1950. 0 U. T. from a given date.

DATE:
: DATE is a subroutine which finds the date from a given time in seconds after 1950. 0 U. T.

LOCATE:
: LOCATE is a subroutine which takes the orbital parameters for a body in an orbit, and the time, and finds the position, angles, velocities etc. of the body.

ORDER:
: ORDER is a program which lists the asteroids and comets on the ephemeris tape in order of increasing semi-major axis, eccentricity, inclination, period, perihelion distance or time of perihelion.

PLDELV:
: PLDELV is a program which calculates the velocity increment required to transfer a spacecraft, approaching a planet in a hyperbolic orbit with respect to the planet, into an elliptical orbit around the planet.

EPHEMERIS
TAPE:
: The ephemeris tape is a BCD FORTRAN II tape with 2 records for each object on the tape. The ephemeris tape concurrently has positional data for the Sun and planets, 87 periodic comets and 1563 asteroids. These data are a set of 7 orbital parameters describing the orbits of the bodies; all orbits on the tape are elliptical although there is no reason why hyperbolic or parabolic orbits could not be added. For the comets and asteroids the orbital parameters are heliocentric, ecliptic, equinox of 1950. 0 and constant; for the planets the parameters are heliocentric, ecliptic, equinox of date and varying with time.

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III.   UNITS, ASSUMPTIONS, ACCURACY

The basic system of units is kilograms - kilometers - seconds of time - radians. All calculations are done in heliocentric, ecliptic, equinox of 1950.0 coordinates; for Cape Canaveral launch restrictions and transformations of planetary ephemerides data the equinox is assumed to rotate at 50.2714 seconds of arc per year. Zero seconds of time corresponds to 1950.0 U. T.

The basic assumptions are

(1) one gravitating body at a time
(2) unperturbed conic section trajectories for all bodies
(3) impulsive thrusting.

The accuracy of the system has not been determined in detail; comparison of Jupiter position calculations with the table "Planetary Coordinates for the Years 1960-1980", H. M. Nautical Almanac Office revealed the following:

For R, the Jupiter-Sun distance, Error Range $= \pm .004\,AU = \pm 0.1\%$

For λ, the Jupiter longitude, Error Range $= -.48$ to 0.56 degrees $= 0.14\%$ of $360°$.

For β, the Jupiter latitude, Error Range $= \pm 0.0020$ degrees $= \pm 0.08\%$ of 2.5 degree range.

Considering the above it is probably safe to assume an overall accuracy of 1 per cent for the system, except when using comet or asteroid trajectories which may not be nearly this accurate. More extensive accuracy checking is planned in the future, and steps are being undertaken to reduce the above errors.

IV. JUPITER CALCULATIONS AND CURVES

1. List of Figures

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Figure 30 ΔV, Applied at Perijove, Necessary to Change Spacecraft Orbit from Hyperbolic to Elliptical with Respect to Jupiter vs R_p for Elliptical Orbits with Various R_a and $V_{SJ} = 10 \, \text{km/sec}$.

Figure 31 ΔV, Applied at Perijove, Necessary to Change Spacecraft Orbit from Hyperbolic to Elliptical with Respect to Jupiter vs R_p for Elliptical Orbits with Various R_a and $V_{SJ} = 15 \, \text{km/sec}$.

Figure 32   Earth, Jupiter, Spacecraft Positions for typical 500 Day Flight, Launch Feb. 8, 1971.

2.   Descriptions of Figures

Figures 1 through 3 illustrate the basic position date used in the calculations; the slight systematic errors in λ, the Earth and Jupiter longitudes shown in Figure 1, are due to small errors in obtaining perihelion times. These times will be adjusted in the near future. These errors (~ 0.7 days for earth, ~ 5.5 days for Jupiter) are too small to affect the subsequent calculations significantly, especially since they are in the same direction for both Earth and Jupiter. At worst they might shift the launch curves in time by a day or two. There will always be a small amount of error in the elliptical representation at the orbits due to the various perturbations. The errors shown in Figures 2 and 3 for Jupiter latitude and radius are also too small to be of any significance; presumably these are a combination of the perihelion time errors and the deviations of the orbit from ellipticity due to the various perturbations.

Figure 4 shows spacecraft weight vs. V_{SE} for Saturn C5, Saturn C1B and Atlas-Centaur. It is included to aid in the interpretation of the subsequent figures and is based on the assumption of a 3rd stage with an ISP of 440 sec and 10 per cent initial stage weight in structures etc.

Figures 5 and 6 are a distillation of Figures 7 through 20; they show (1) the minimum V_{SE} required to be able to launch anytime in 30 day launch windows from 1966 to 1977, for various times of flight, and (2) the minimum V_{SE} in each launch window. Note that $\overline{V}_{SE} = \overline{V}_S - \overline{V}_E = \sqrt{100\,C_3}$, km/sec, where $\overline{V}_S$ = spacecraft velocity after earth escape, $\overline{V}_E$ = earth velocity, C_3 = JPL twice the injection kinetic energy/unit mass with units $m^2/sec^6 \times 10^8$. Note that the actual minimum V_{SE} in any launch window will be significantly smaller than the V_{SE} required for launch anytime in a 30 day period.

Figure 6 shows the expected decrease in minimum required V_{SE} for increasing times of flight, and the differences between various launch windows. Figure 5 is a more interesting curve and of greater practical importance; note that for the 1972-73-74 launch windows, if we require high enough V_{SE} for a 30 day launch period, it actually requires considerably less energy, V_{SE}, to perform a 500 day mission than to perform an 800 day mission. In fact the minimum V_{SE} shown on Figure 6 is 7.8, while the minimum V_{SE} for launch anytime in a 30 day launch window, shown in Figure 5, is 10.3, a very significant increase in V_{SE} of 2.5 km/sec due to launch window requirements. Figure 13 shows the reason for this: for the 800 day, 1973 flight window the angle, earth-sun-Jupiter at arrival is close to 180° at near the optimum launch time; since the 180° flight requires a highly inclined orbit it is a high energy mission (the spacecraft must "fly over the Sun"); the high energy spike shown in the figure narrows the width of the favorable launch times and pushes up the V_{SE} for a launch anytime within 30 days. The following explanation and examples should make this clearer.

Consider the minimum energy Hohmann transfer from one body through an angle DU = 180° to the second body, both bodies traveling in the same direction in the same plane in circular orbits. In the real case of Earth-Jupiter the approximately circular, approximately co-planar orbits will have minima for the longer (greater than 600 day) flight times near the 180° condition; however it is a physical requirement that the earth-sun-Jupiter at arrival points be in a single plane for non-thrusted trajectories. Thus if the earth-Jupiter at arrival angle is nearly 180° in longitude but Jupiter is out of the ecliptic plane, a plane passing through the earth-sun-Jupiter at arrival points will be highly inclined to the ecliptic and the velocity required for this type of trajectory will be high. The following table illustrates this for 800 day flights near the optimum 1973 launch.

Date of Launch	DU	V_{SE}	i
3/19/73	188. 5°	11. 1 km/sec	8. 6°
3/24/73	184. 5°	13. 9 km/sec	17. 5°
3/29/73	178. 8°	42. 3 km/sec	75. 2°
4/3/73	174. 8°	12. 5 km/sec	15. 2°
4/8/73	170. 7°	9. 9 km/sec	8. 0°

DU is the angle earth at departure-sun-Jupiter at arrival, V_{SE} is the velocity increment required for the flight, i is the inclination of the flight plane to the ecliptic plane. Jupiter at time of arrival of the spacecraft is about 1 degree below the ecliptic plane in all cases.

Figures 7, 8, 9, and 10 show V_{SE} for 200, 300, 400, 500 day flights every 5 days through the 1966 to 1977 launch windows. Figures 11, 12, 13 and 14 show the same type of data for 600, 800, and 1000 day times

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of flight. It is interesting to note that the problem of "launching over the Sun" shows clearly in these figures, and is most severe as stated earlier for the longer times of flight where the most favorable earth-sun-Jupiter at arrival angles approach 180°.

Figures 15 through 20 show the V_{SE} required throughout the 1966 to 1978 time periods for 200 through 1000 day earth to Jupiter flights.

Figure 21 shows the earth to Jupiter (communications) distance at time of arrival of spacecraft at Jupiter for launches in the launch windows. It is interesting to note that the communications distance and thus the communications problem from spacecraft to earth will be smallest for 500 day flights.[*] However, due to the direct line of sight appearance of the Sun behind the Earth, as seen from the spacecraft, the 500 day flight communications from Earth to spacecraft will take place with the high solar noise source prominent in the background.

Figures 22 through 25 show the spacecraft velocity of approach to Jupiter, V_{SJ} for various times of flight in the 1966 to 1977 launch window. Note that $V_{SJ} = \vec{V}_S - \vec{V}_J = V_{\infty}$ where $\vec{V}_S$ = spacecraft velocity at intersection of its trajectory with Jupiters trajectory, i. e. unperturbed by the Jovian gravitational field, and $\vec{V}_J$ = Jovian velocity in orbit at the same time.

Figure 26 defines the geometry of the near Jupiter trajectory. The asymptotic miss distance is the distance of closest approach to the Jupiter ignoring the Jovian gravitational attraction on the spacecraft. R_p is the perijove distance, the closest approach of the spacecraft to Jupiter

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[*] The earth-Jupiter of spacecraft arrival distance is relatively constant for any time of flight due to the cyclic nature of the launch window geometries; for 500 day times of flight the sun-earth-Jupiter positions are approximately on a straight line.

considering the Jovian gravitational attraction; when the spacecraft is at perijove a thrust is applied to change its orbit from hyperbolic to elliptical with respect to Jupiter.

Figure 27 shows the perijove distance as a function of the asymptotic miss distance for various values of V_{SJ}. Figure 28 then shows the ΔV which must be applied at perijove to change the spacecraft's hyperbolic trajectory with respect to Jupiter into a specific elliptical trajectory around Jupiter, namely one for which R_p is 3 Jupiter Radii, R_a the apojove distance is 100 Jupiter Radii and the period is 44 (earth) days.

Figures 29 through 31 show the ΔV which must be applied at perijove to put the spacecraft into various elliptical orbits around Jupiter, for V_{SJ} is of 5, 10 and 15 km/sec respectively.

Figure 32 shows the positions of earth-Jupiter and the spacecraft throughout a typical 500 day flight.
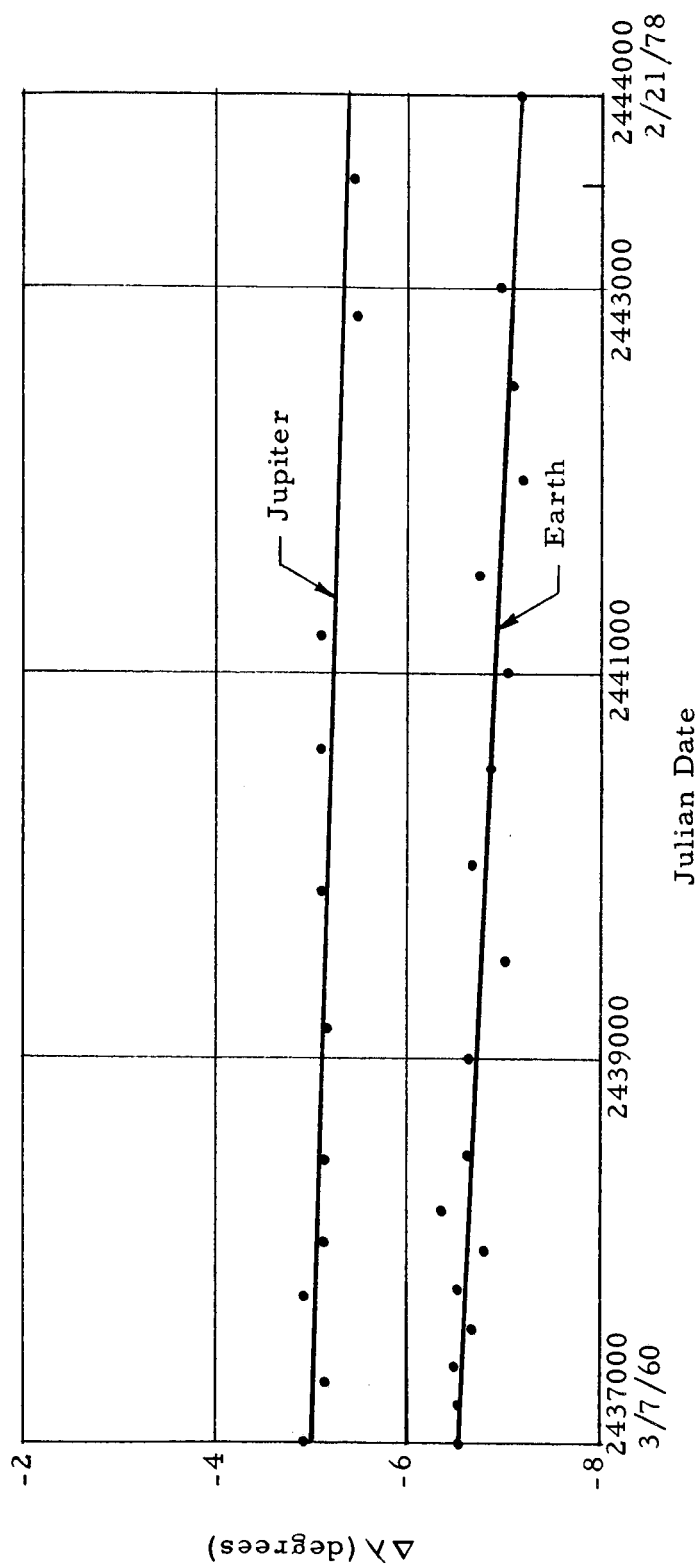
Fig. 1   "ACTUAL" - 7090 CALCULATED LONGITUDES OF JUPITER AND EARTH
VS DATE, 1960 - 1980
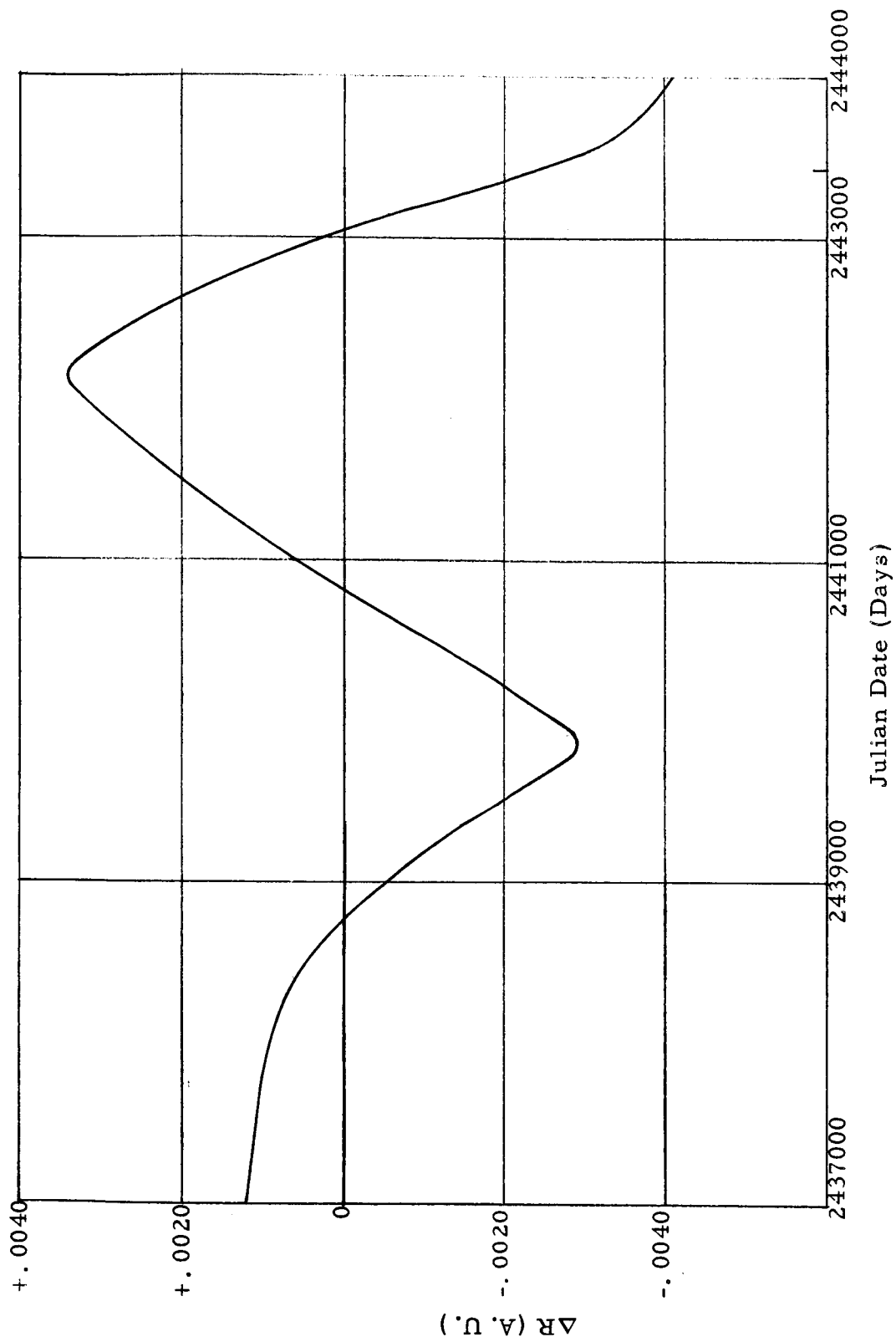
Fig. 2   "ACTUAL" - 7090 CALCULATED JUPITER - SUN DISTANCE VS DATE, 1960 - 1980
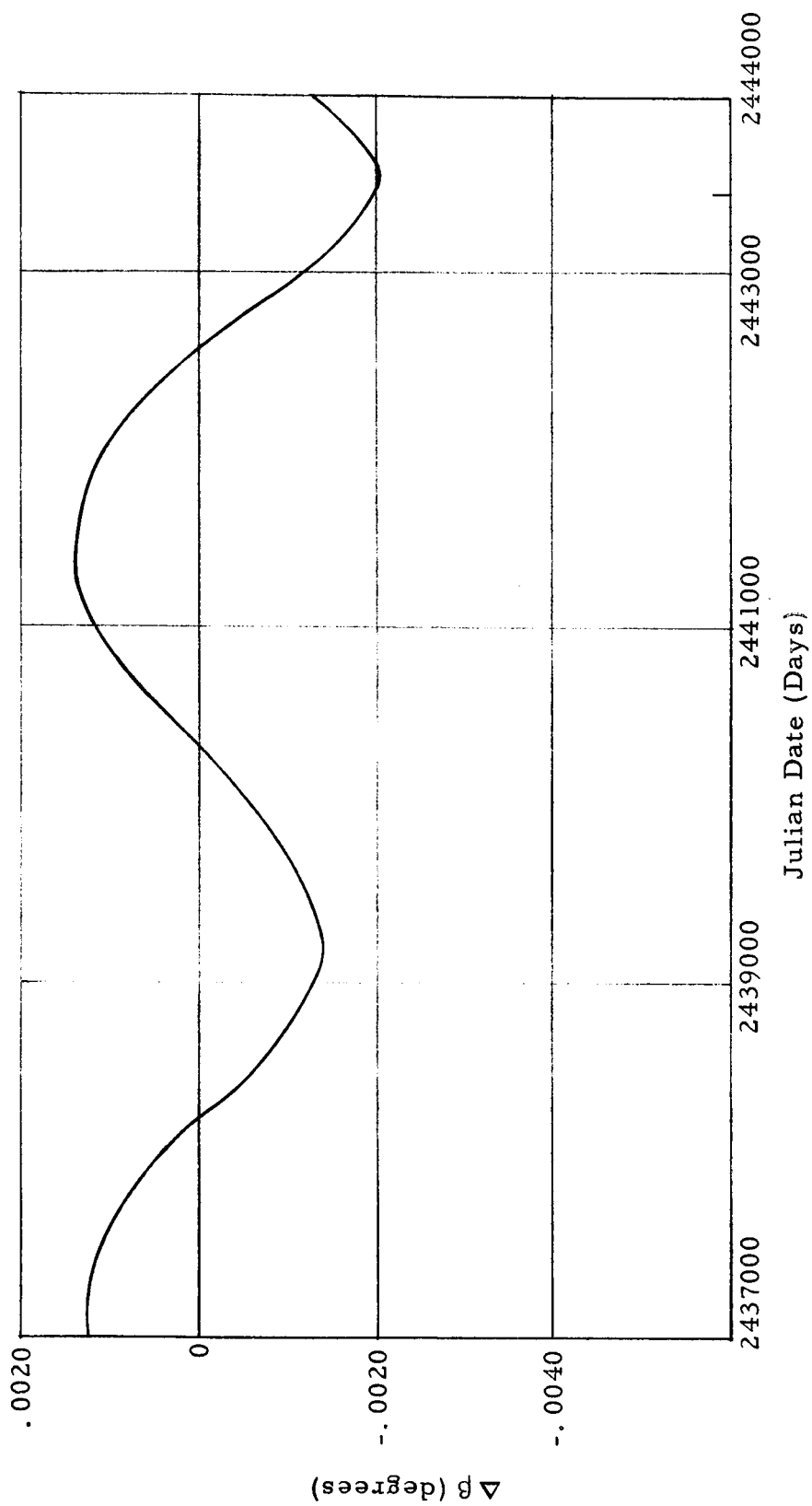
Fig. 3   "ACTUAL" 7090 CALCULATED LATITUDE OF JUPITER VS. DATE, 1960-1980
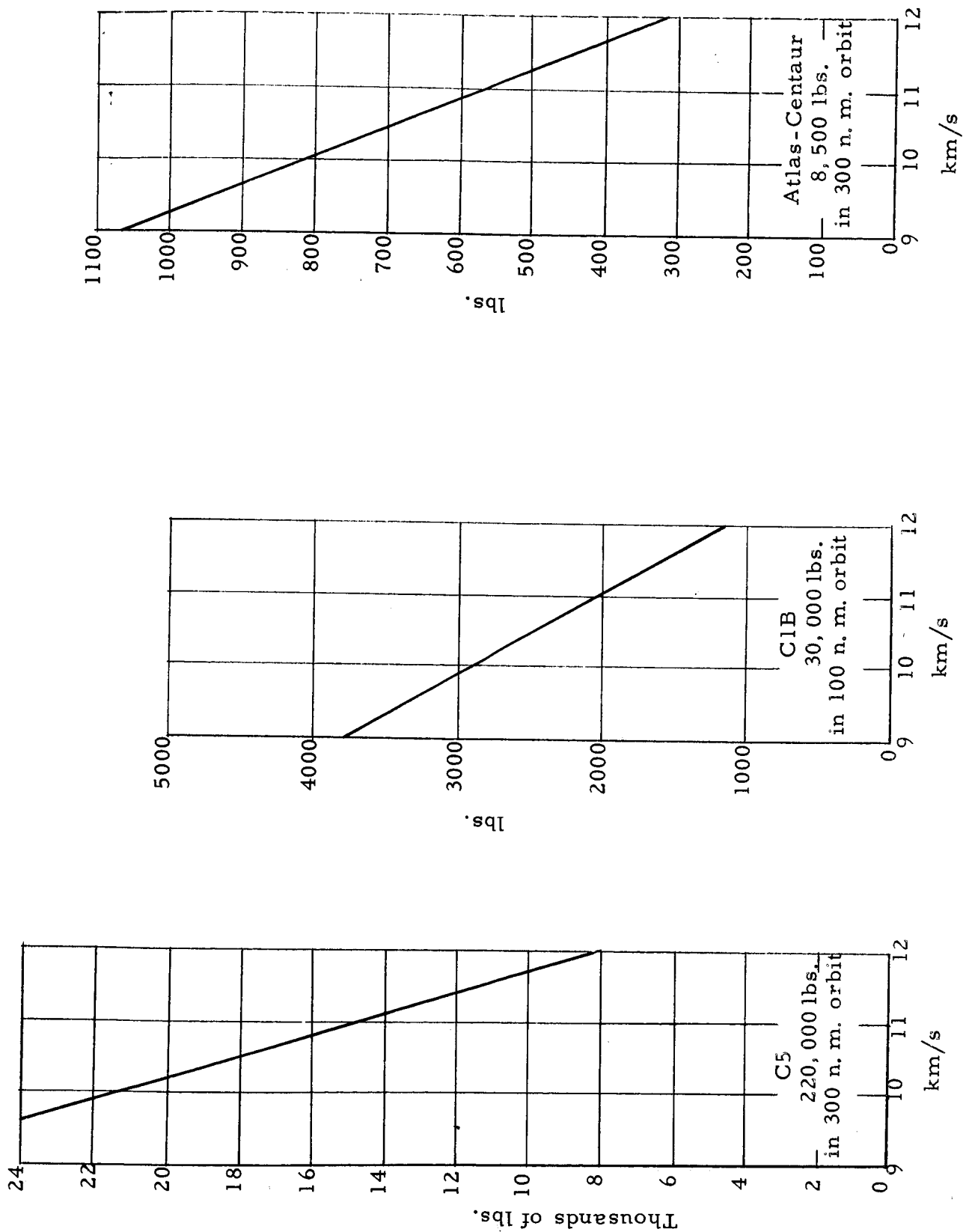
Fig. 4   TOTAL SPACECRAFT WEIGHT VS V_{SE} FOR THREE LAUNCH VEHICLES
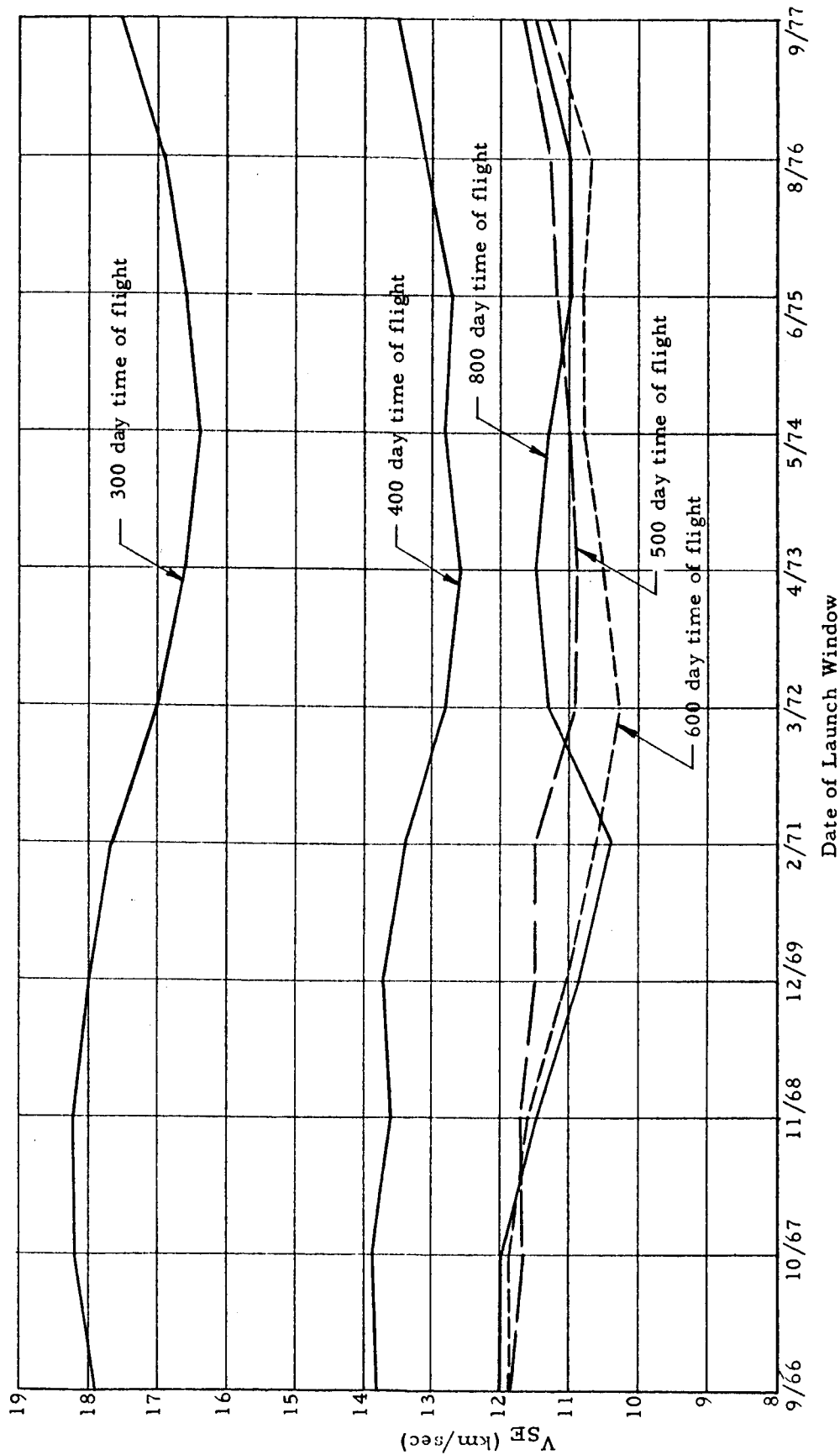
Fig. 5   V$_{SE}$ REQUIRED FOR LAUNCH ANYTIME IN 30 DAY LAUNCH WINDOW, EARTH TO JUPITER, 1966 TO 1977 LAUNCH WINDOWS, FOR 300, 400, 500, 600, 800 DAY TIMES OF FLIGHT
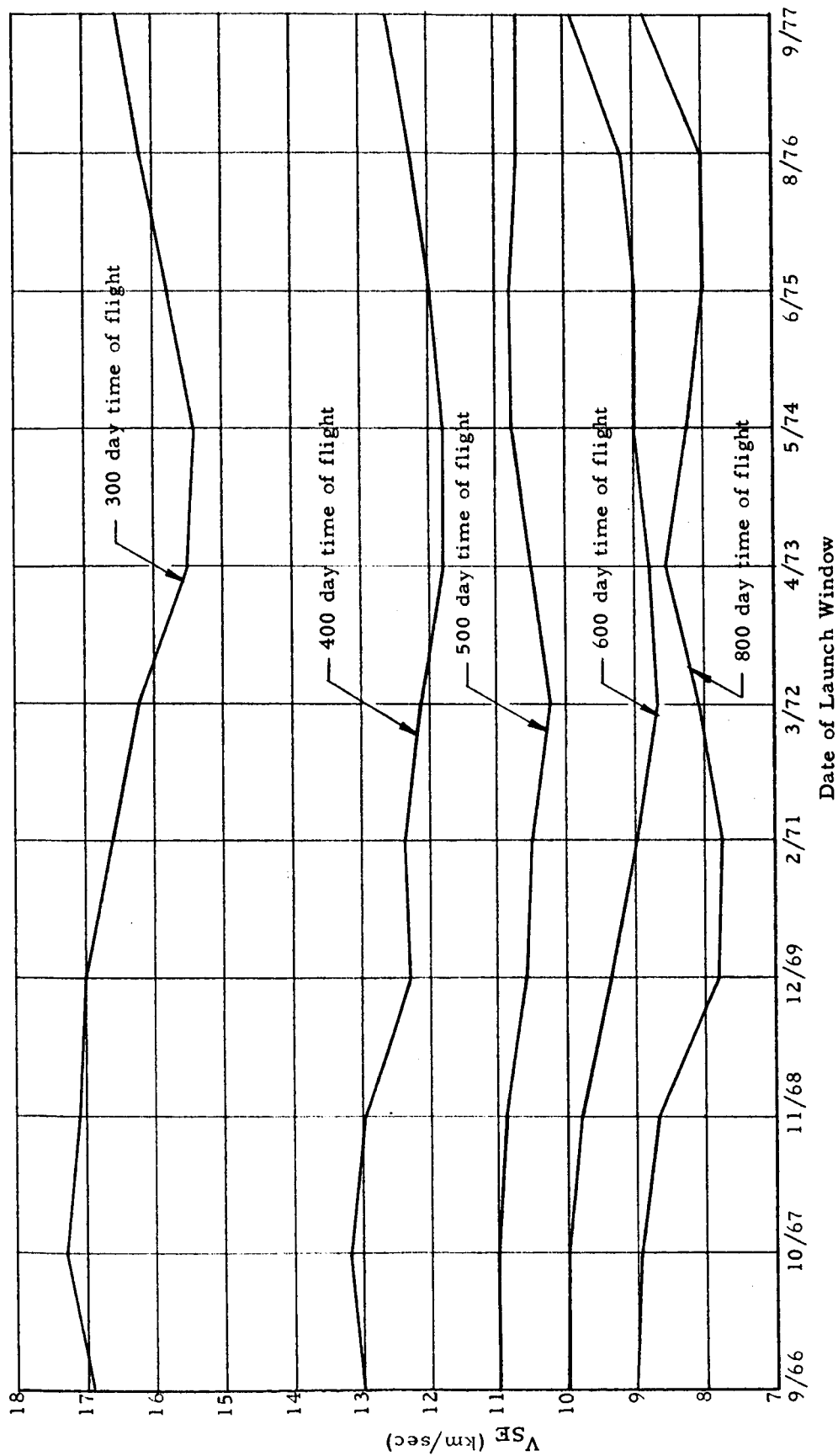
Fig. 6   MINIMUM V_{SE} REQUIRED FOR LAUNCH, EARTH TO JUPITER, 1966-1977 LAUNCH WINDOWS, FOR 300, 400, 500, 600, 800 DAY TIMES OF FLIGHT
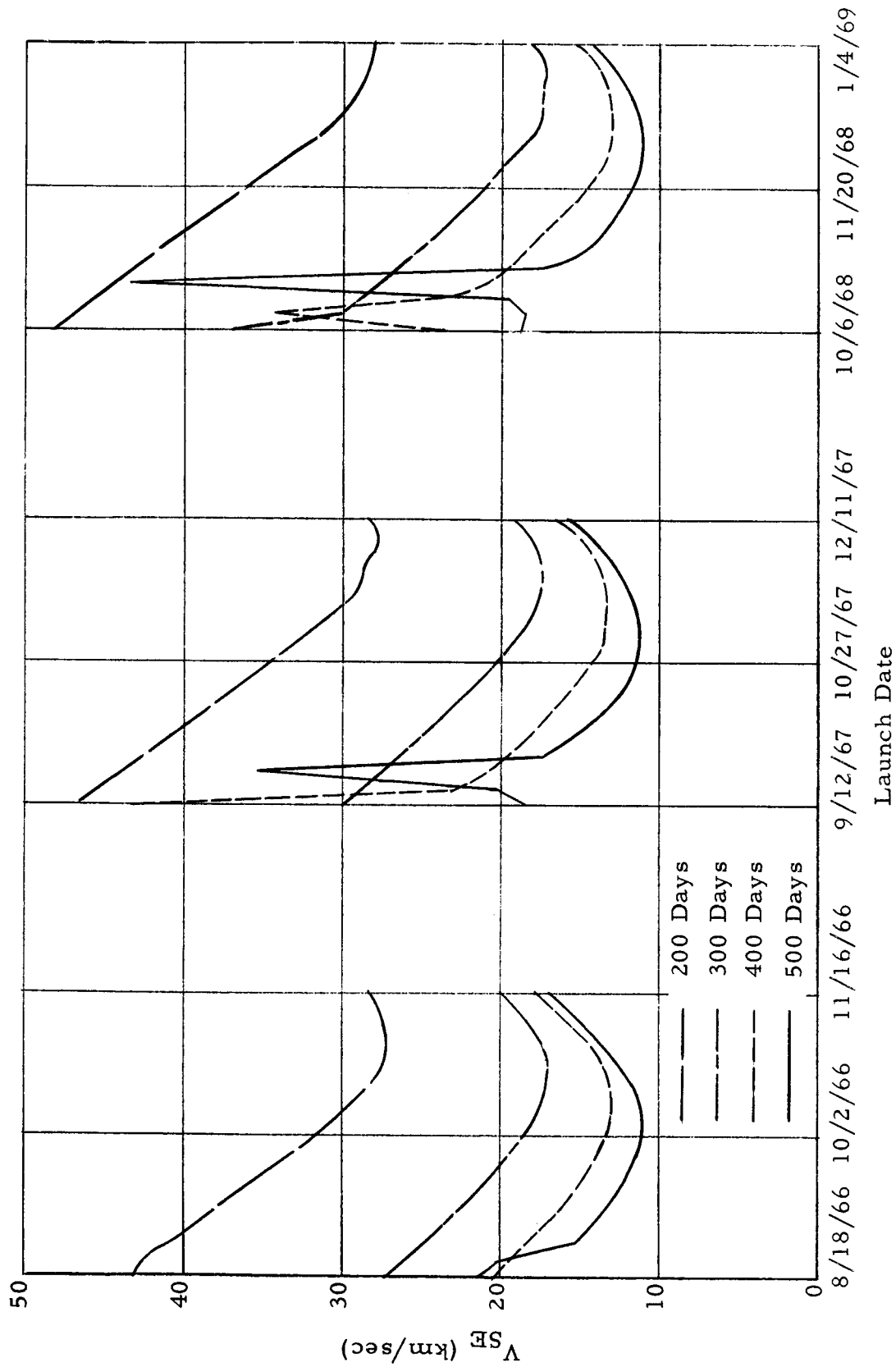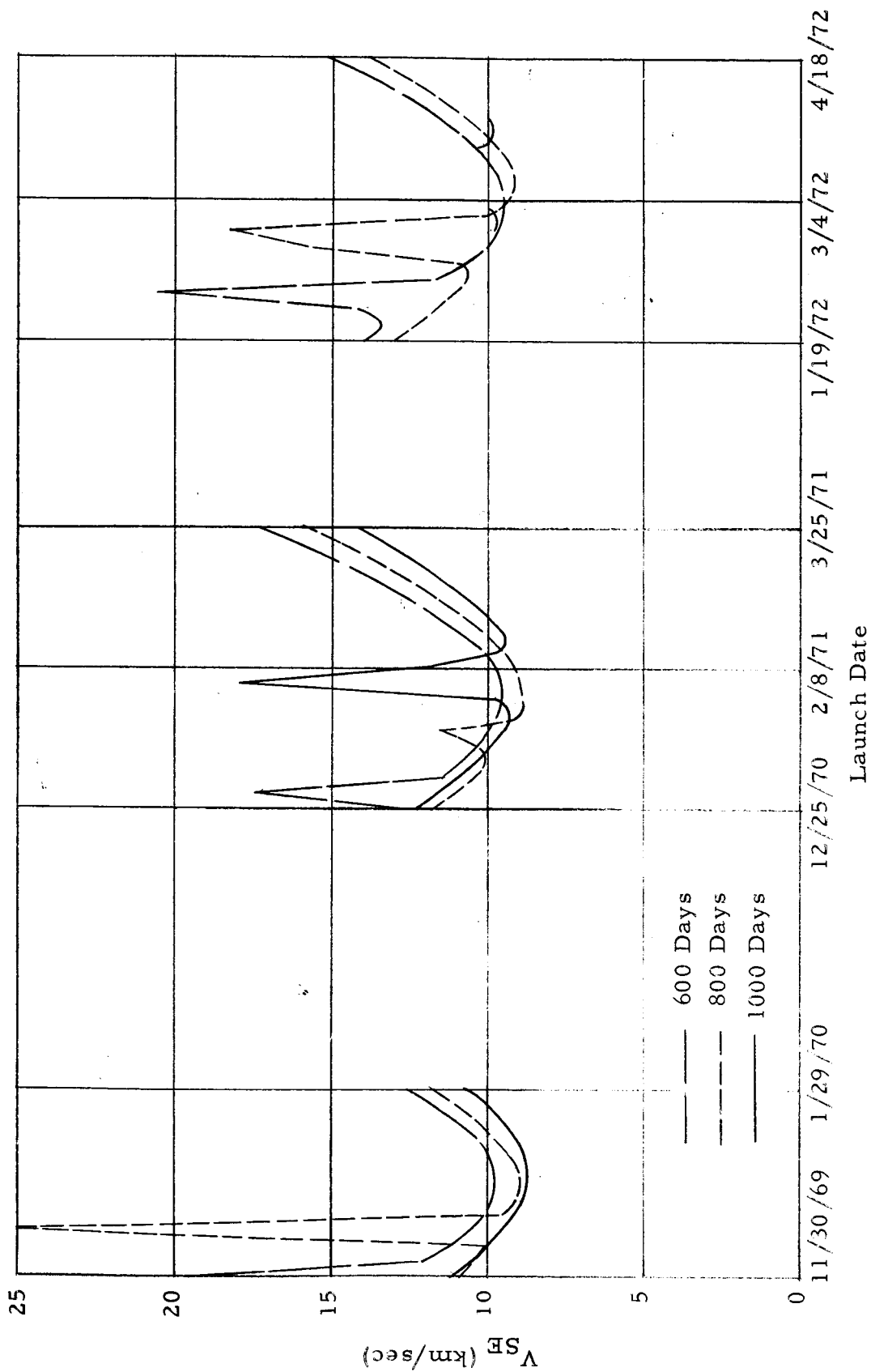
Fig. 7 V_{SE} REQUIRED FOR 200, 300, 400, 500 DAY FLIGHTS, EARTH TO JUPITER LAUNCHES EVERY 5 DAYS IN 1966, 1967 OR 1968 LAUNCH WINDOWS
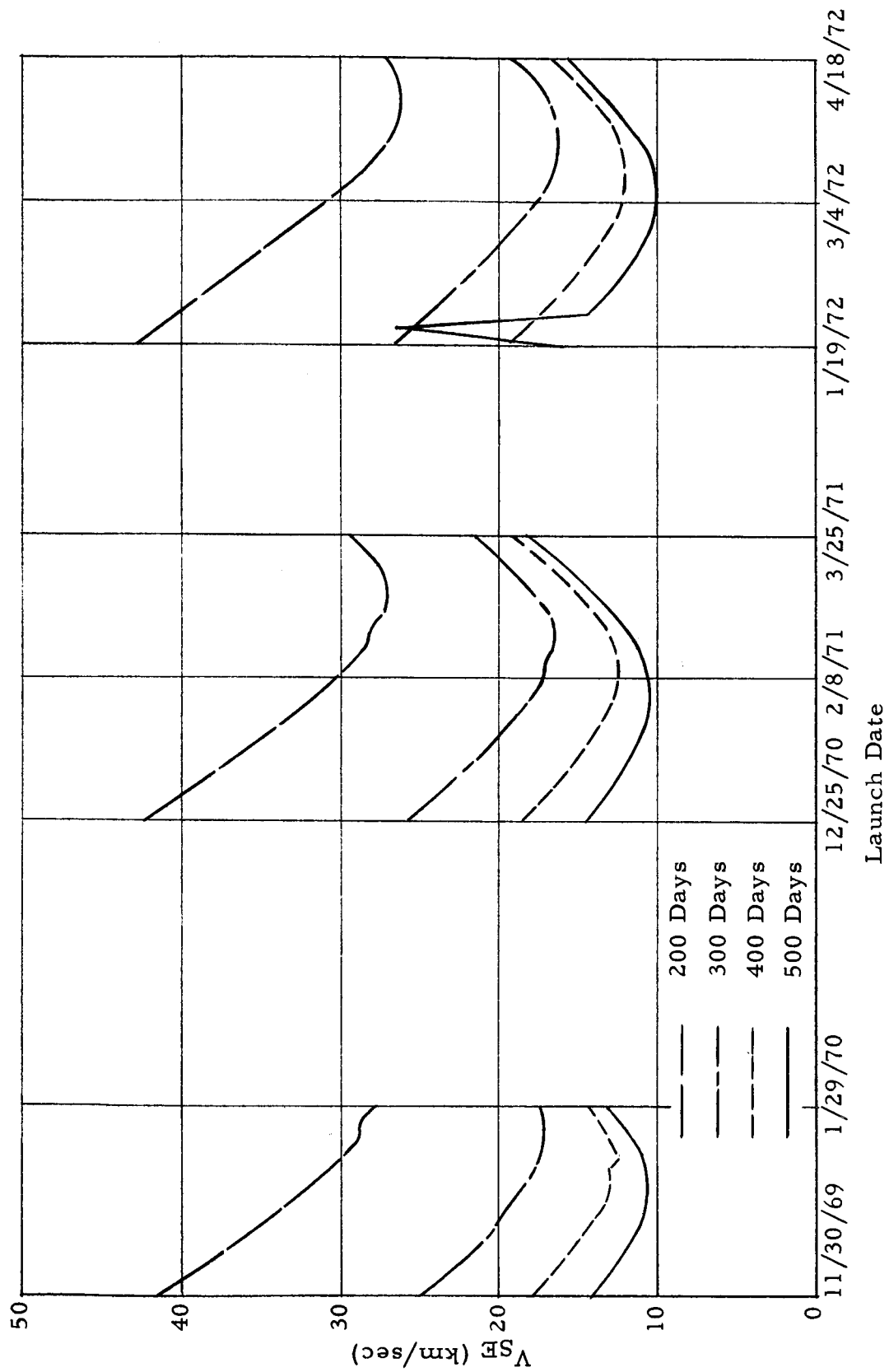
Fig. 8 V_SE REQUIRED FOR 200, 300, 400, 500 DAY FLIGHTS EARTH TO JUPITER
LAUNCHES EVERY 5 DAYS IN 1969, 1971 OR 1972 LAUNCH WINDOWS
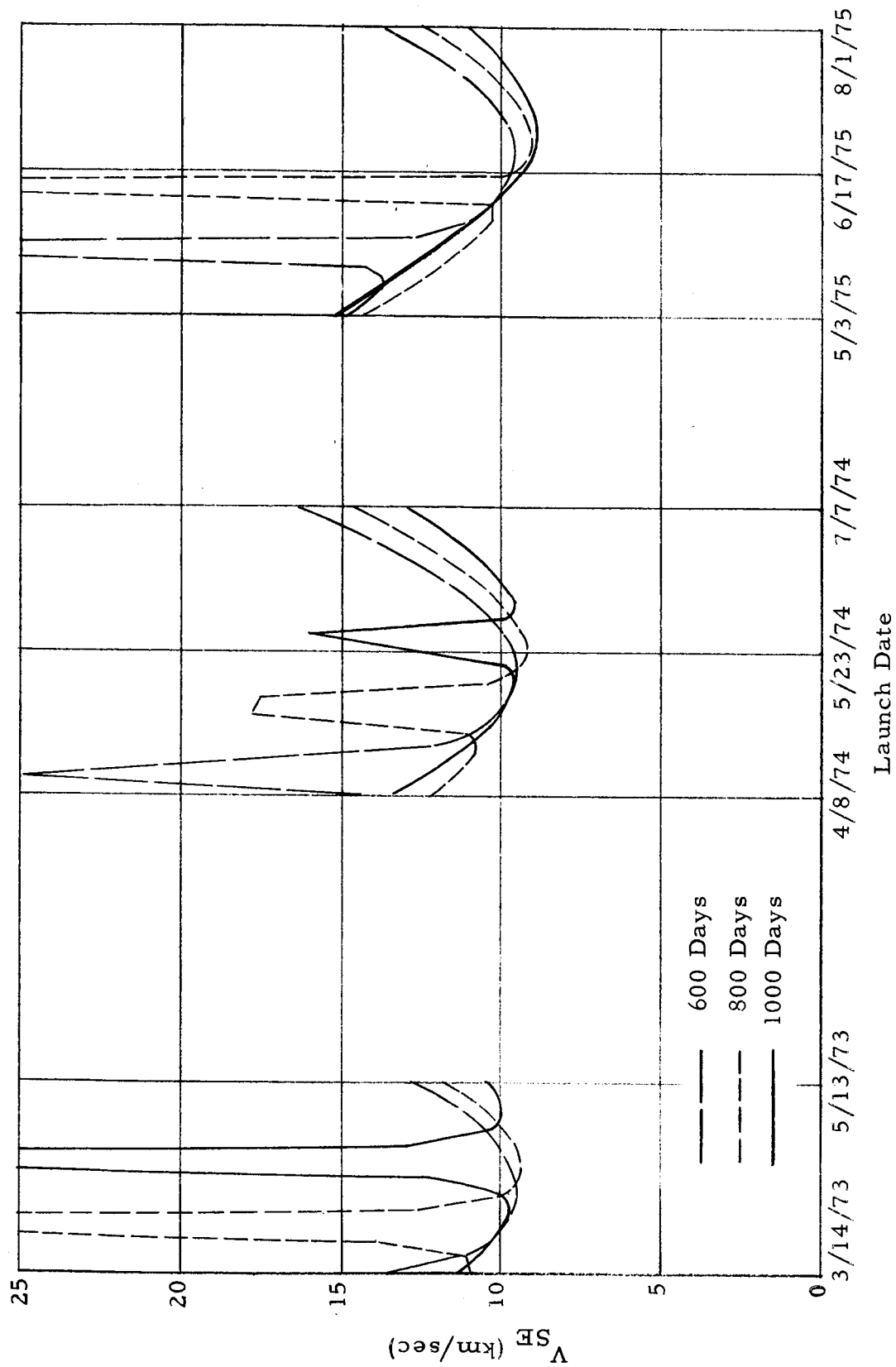
-19-
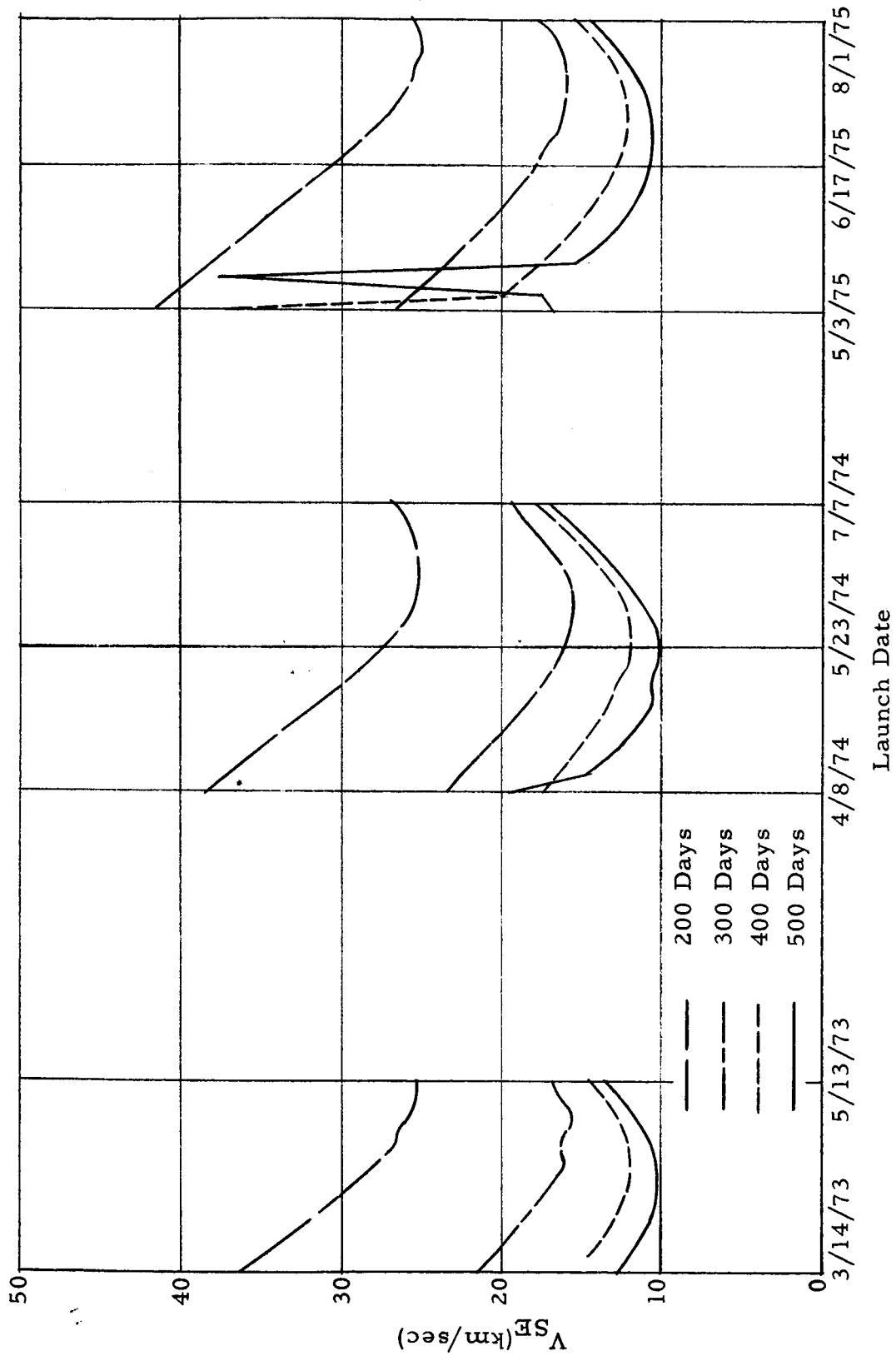
Fig. 9 V_{SE} REQUIRED FOR 200, 300, 400, 500 DAY FLIGHTS EARTH TO JUPITER, LAUNCHES EVERY 5 DAYS IN 1973, 1974 OR 1975 LAUNCH WINDOWS
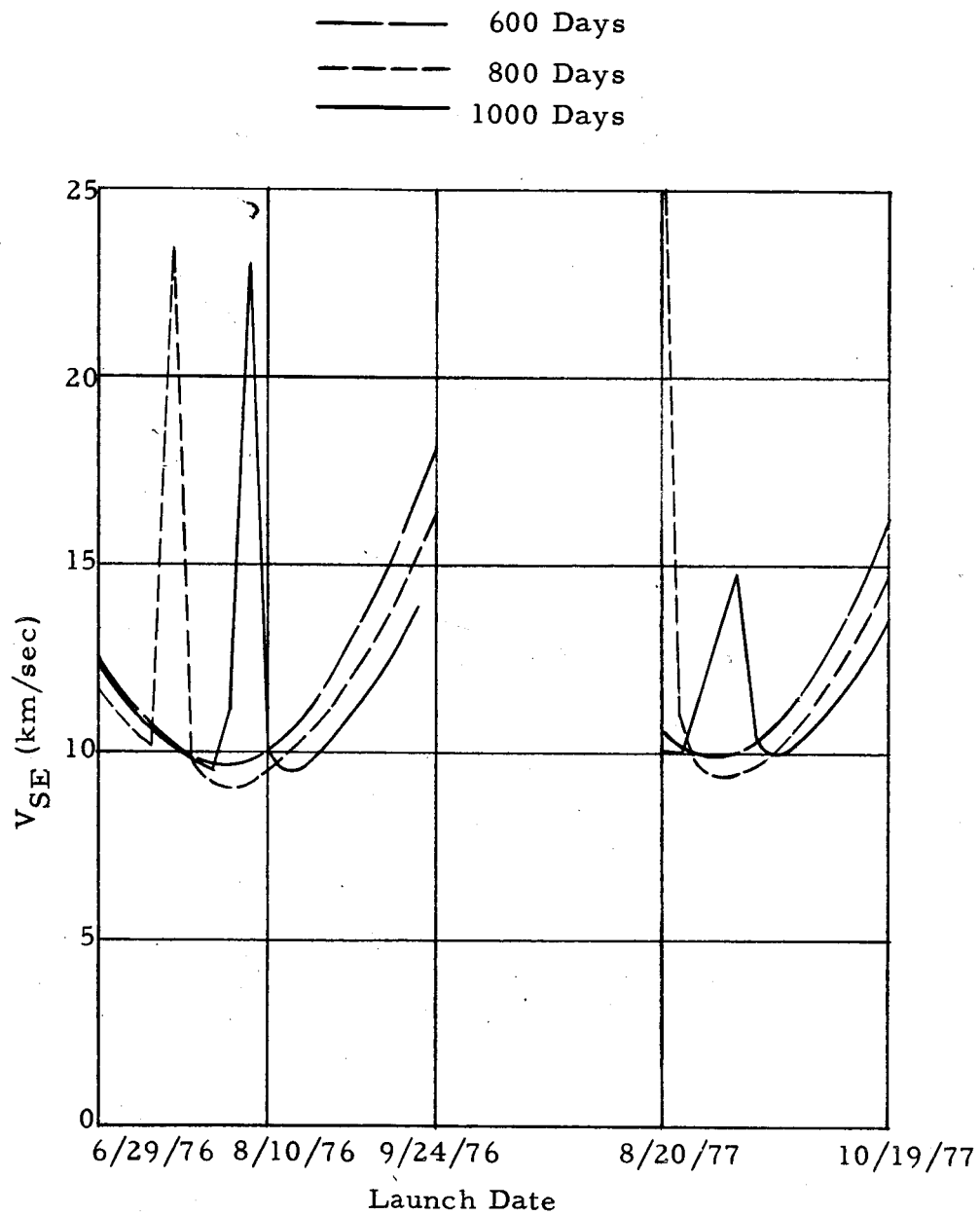
Fig. 10 V_{SE} REQUIRED FOR 200, 300, 400, 500 DAY FLIGHTS EARTH TO JUPITER LAUNCHES EVERY 5 DAYS IN 1976 OR 1977 LAUNCH WINDOWS

Legend:
- 200 Days
- 300 Days
- 400 Days
- 500 Days

Y-axis: V_{SE} (km/sec), values 0, 10, 20, 30, 40

X-axis: Launch Date — 6/26/76 8/10/76 9/24/76 8/20/77 10/19/77
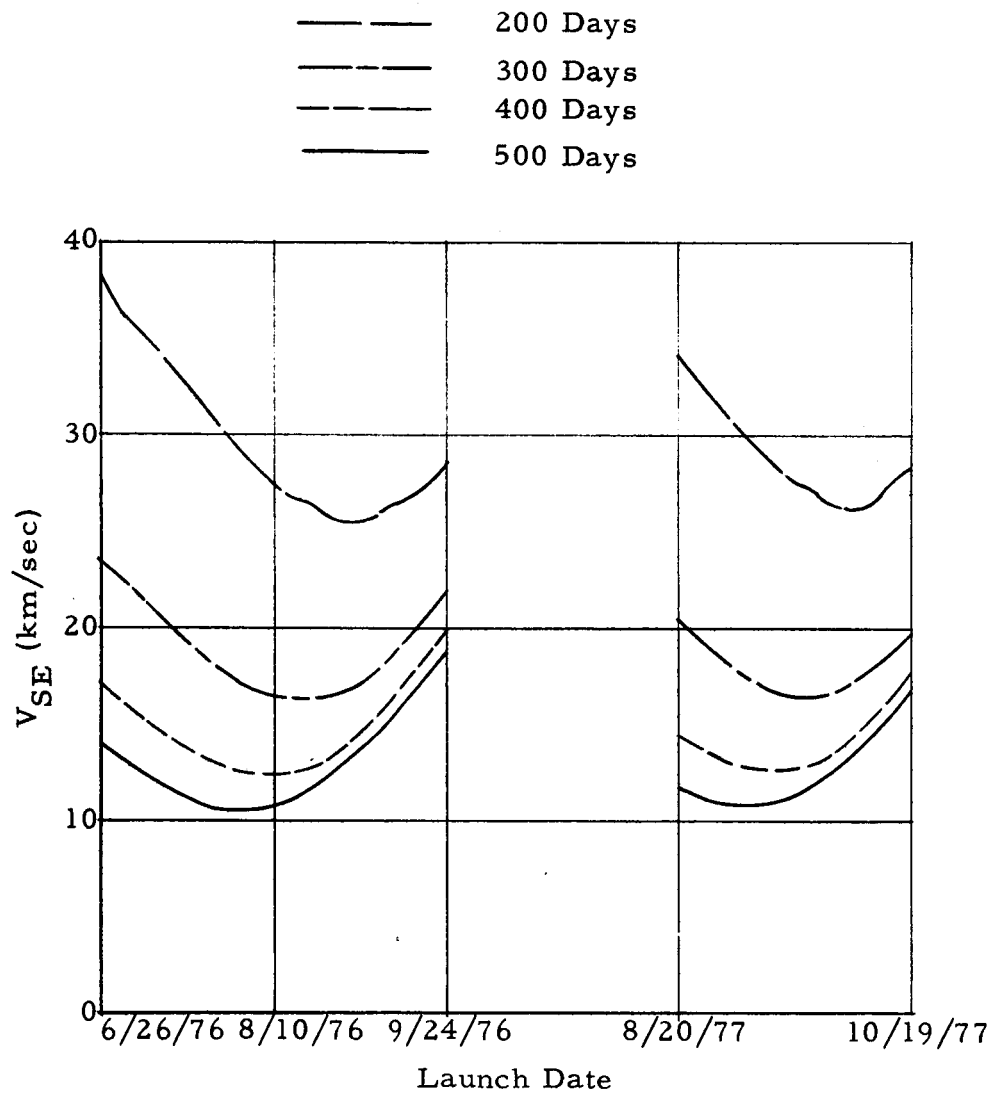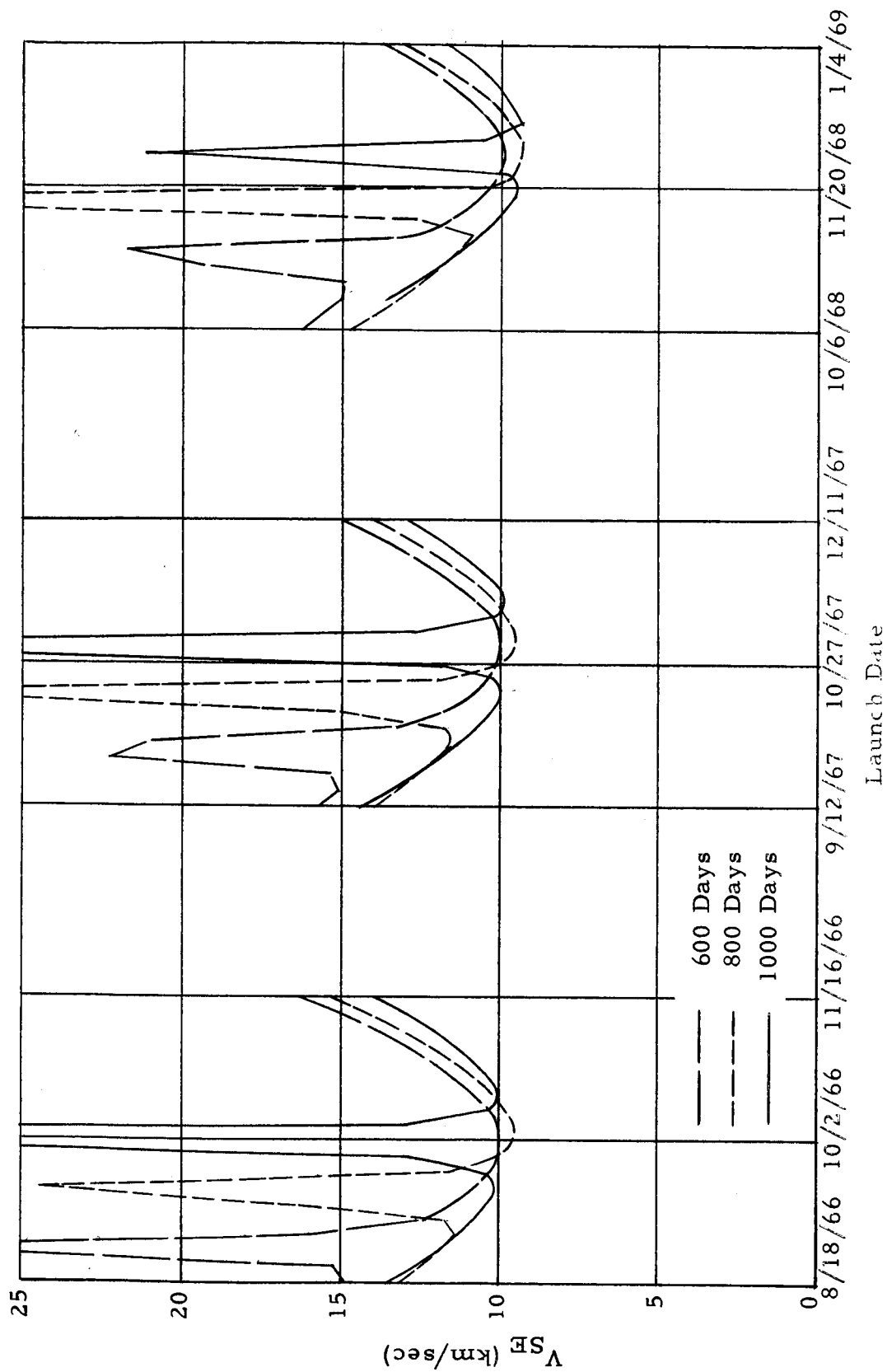
Fig. 11 V_{SE} REQUIRED FOR 500, 800, 1000 DAY FLIGHTS, EARTH TO JUPITER
LAUNCHES EVERY 5 DAYS IN 1966, 1967 OR 1968 LAUNCH WINDOWS

Fig. 12 V_{SE} REQUIRED FOR 600, 800, 1000 DAY FLIGHTS, EARTH TO JUPITER LAUNCHES EVERY 5 DAYS IN 1969, 1971 OR 1972 LAUNCH WINDOWS

Fig. 13 V_{SE} REQUIRED FOR 600, 800, 1000 DAY FLIGHTS, EARTH TO JUPITER LAUNCHES EVERY 5 DAYS IN 1973, 1974 OR 1975 LAUNCH WINDOWS

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Fig. 14 V_{SE} REQUIRED FOR 600, 800, 1000 DAY FLIGHTS, EARTH TO JUPITER LAUNCHES EVERY 5 DAYS IN 1976 OR 1977 LAUNCH WINDOWS
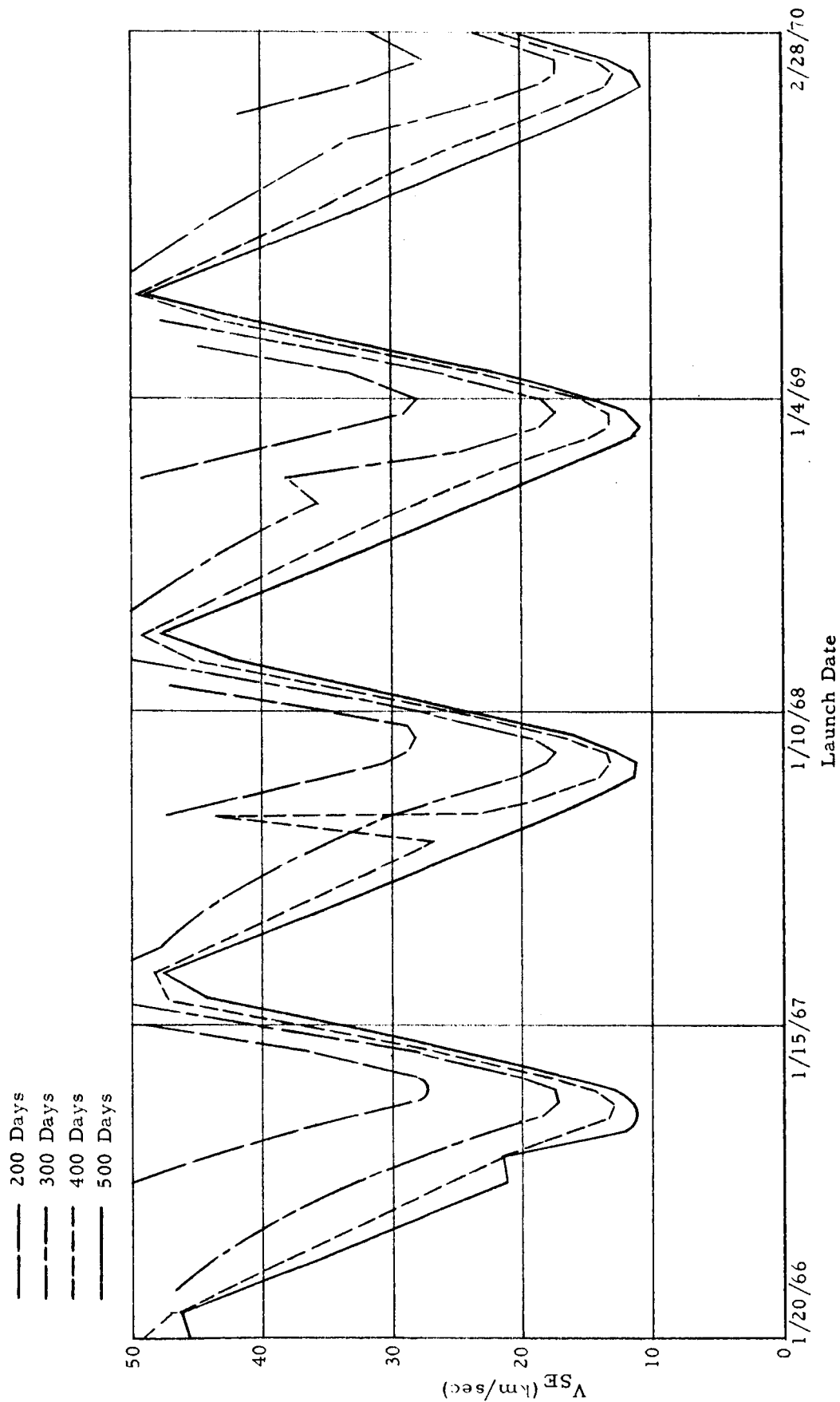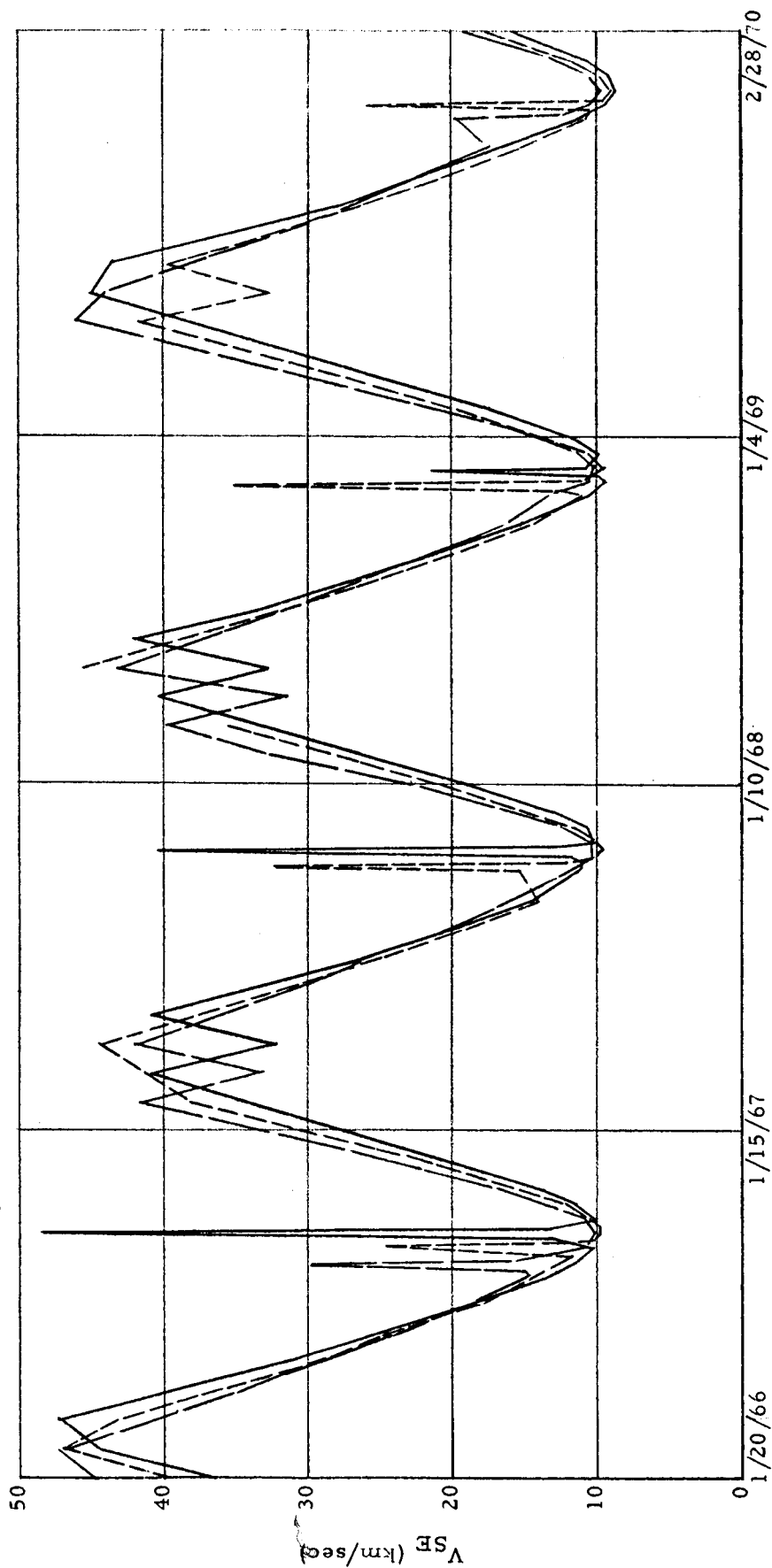
Fig. 15 V_{SE} REQUIRED FOR 200, 300, 400, 500 DAY FLIGHTS, EARTH TO JUPITER, 1966 TO 1970

Fig. 16 V_{SE} REQUIRED FOR 200, 300, 400, 500 DAY FLIGHTS, EARTH TO JUPITER, 1970 TO 1974
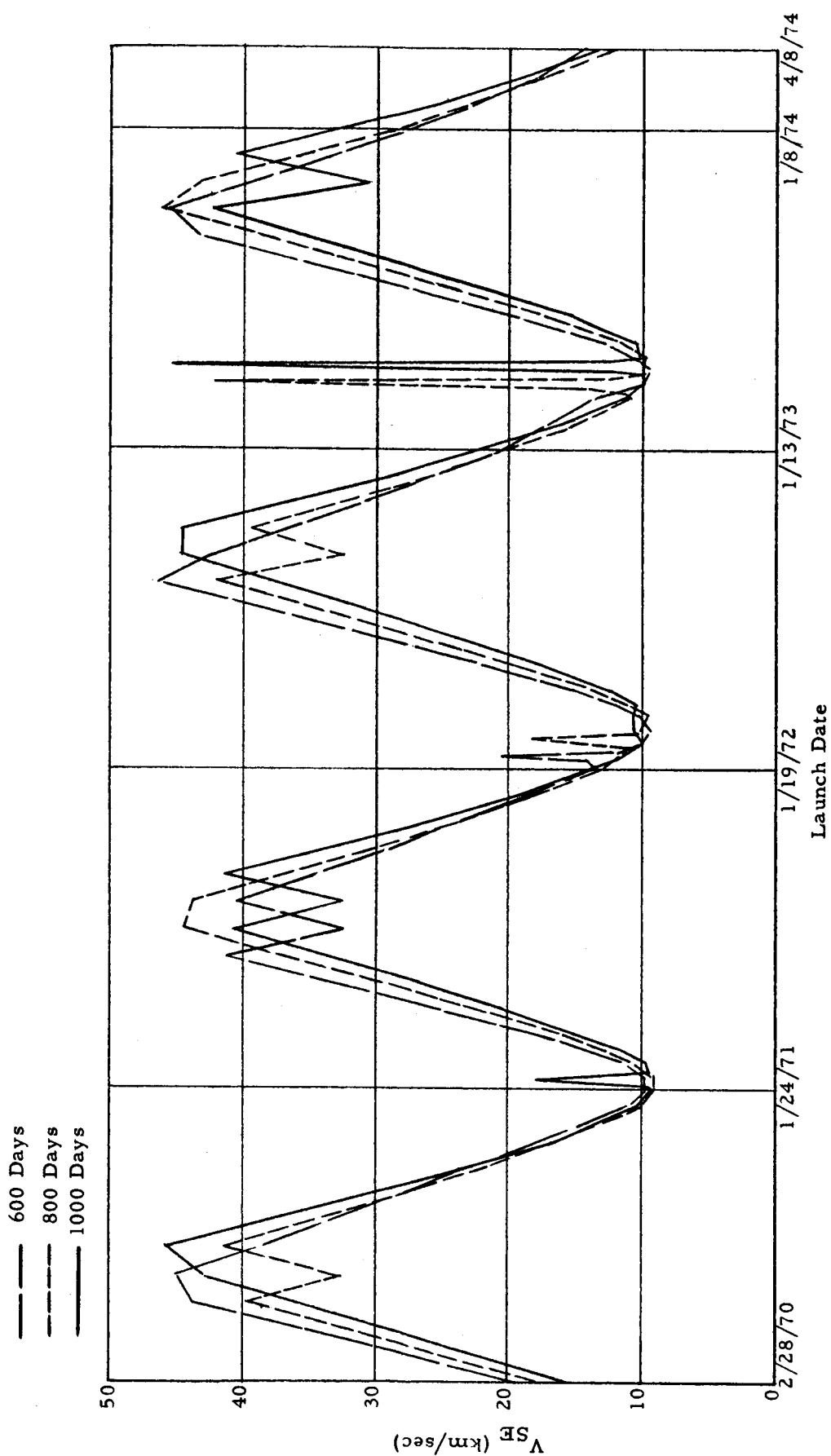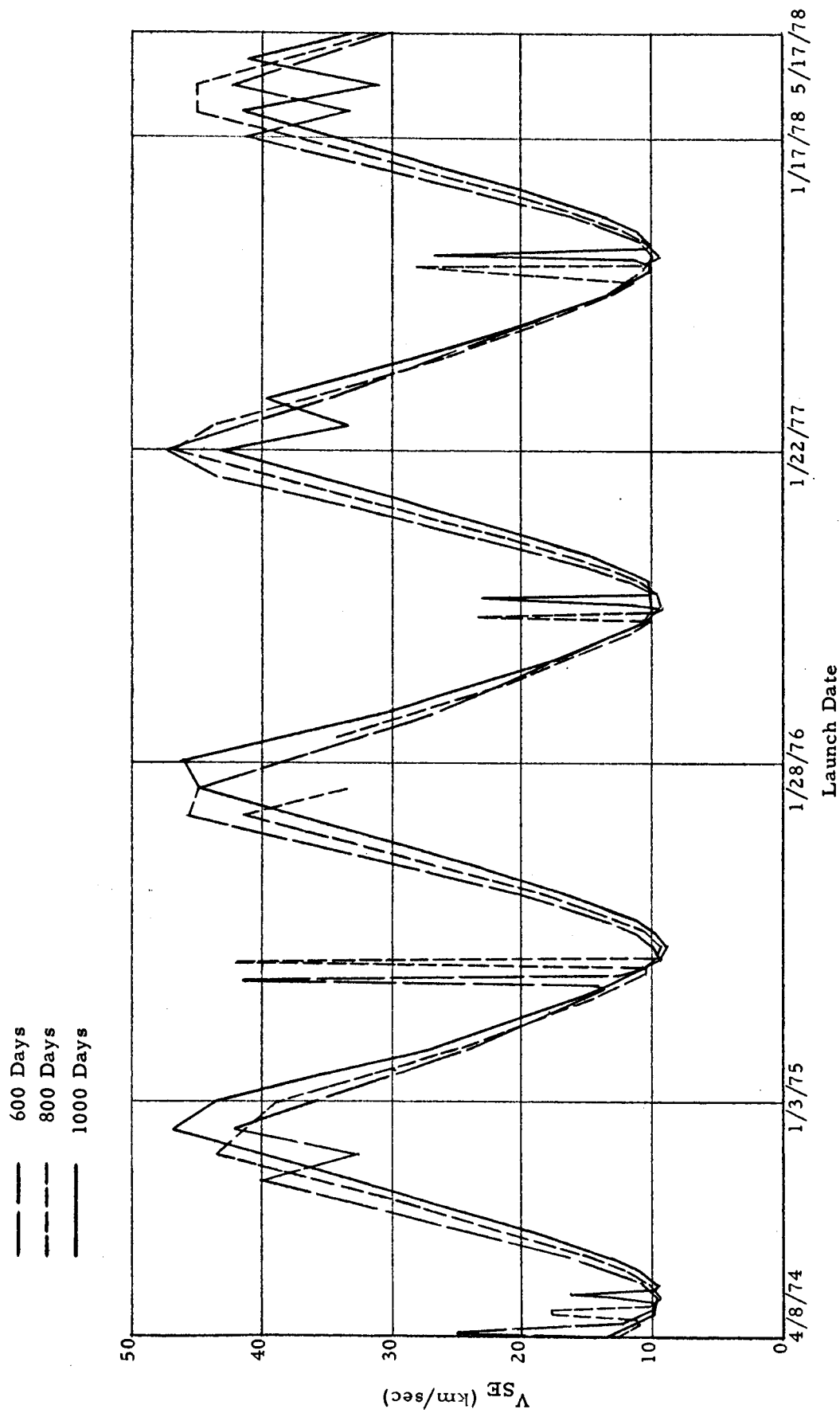
Fig. 17 V_{SE} REQUIRED FOR 200, 300, 400, 500 DAY FLIGHTS, EARTH TO JUPITER, 1974 TO 1978

Fig. 18 V_{SE} REQUIRED FOR 600, 800, 1000 DAY FLIGHTS, EARTH TO JUPITER, 1966 TO 1970

600 Days
800 Days
1000 Days

V_{SE} (km/sec)

Launch Date

1/20/66   1/15/67   1/10/68   1/4/69   2/28/70

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Fig. 19 V_{SE} REQUIRED FOR 600, 800, 1000 DAY FLIGHTS, EARTH TO JUPITER, 1970 TO 1974

Fig. 20 V$_{SE}$ REQUIRED FOR 600, 800, 1000 DAY FLIGHTS, EARTH TO JUPITER, 1974 TO 1978
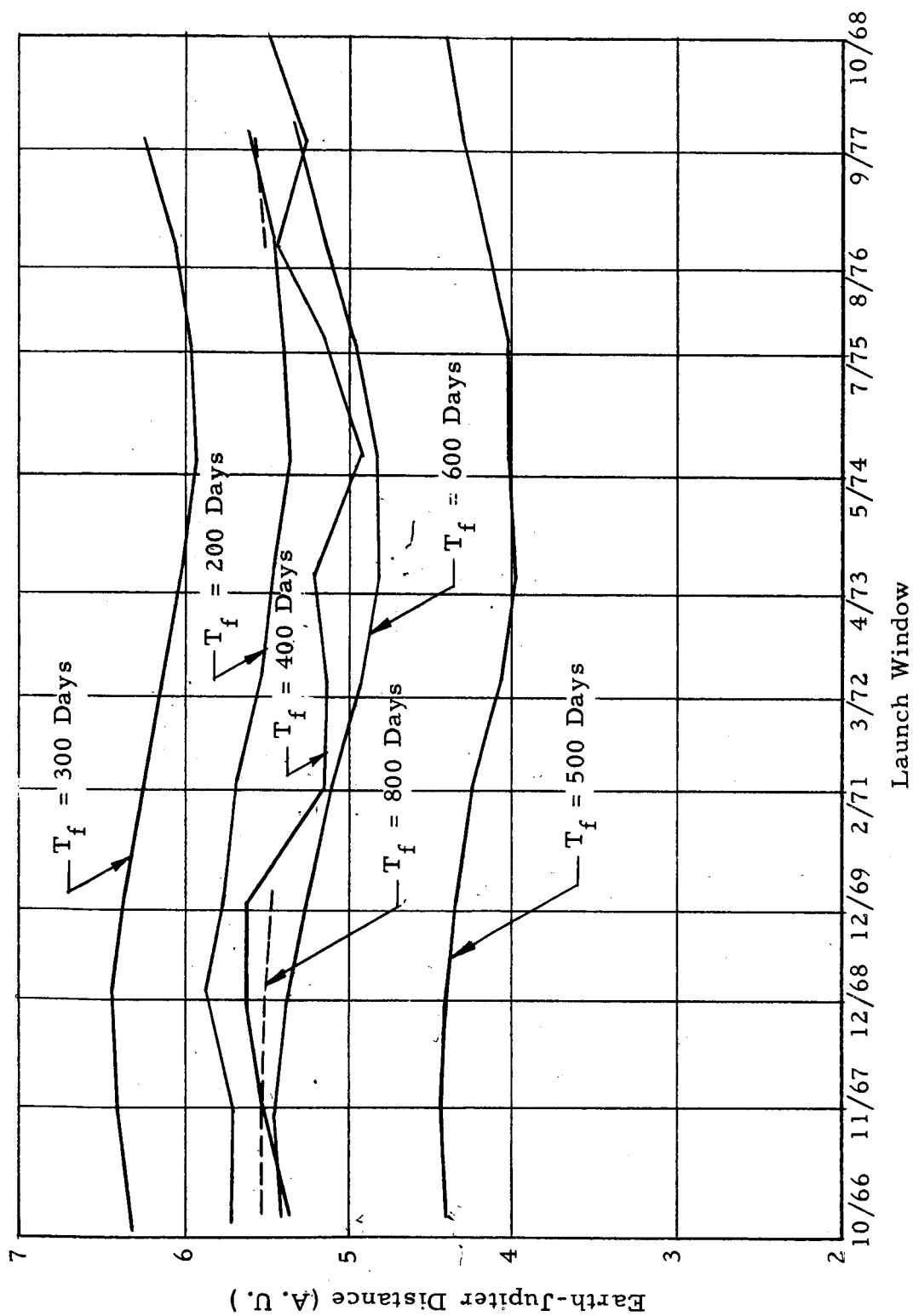
600 Days
800 Days
1000 Days

V$_{SE}$ (km/sec)

Launch Date

4/8/74   1/3/75   1/28/76   1/22/77   1/17/78   5/17/78

Fig. 21   EARTH TO JUPITER (COMMUNICATIONS) DISTANCE AT TIME OF ARRIVAL OF SPACECRAFT AT JUPITER FOR MINIMUM V_{SE} FLIGHTS IN 1966 TO 1978 LAUNCH WINDOWS
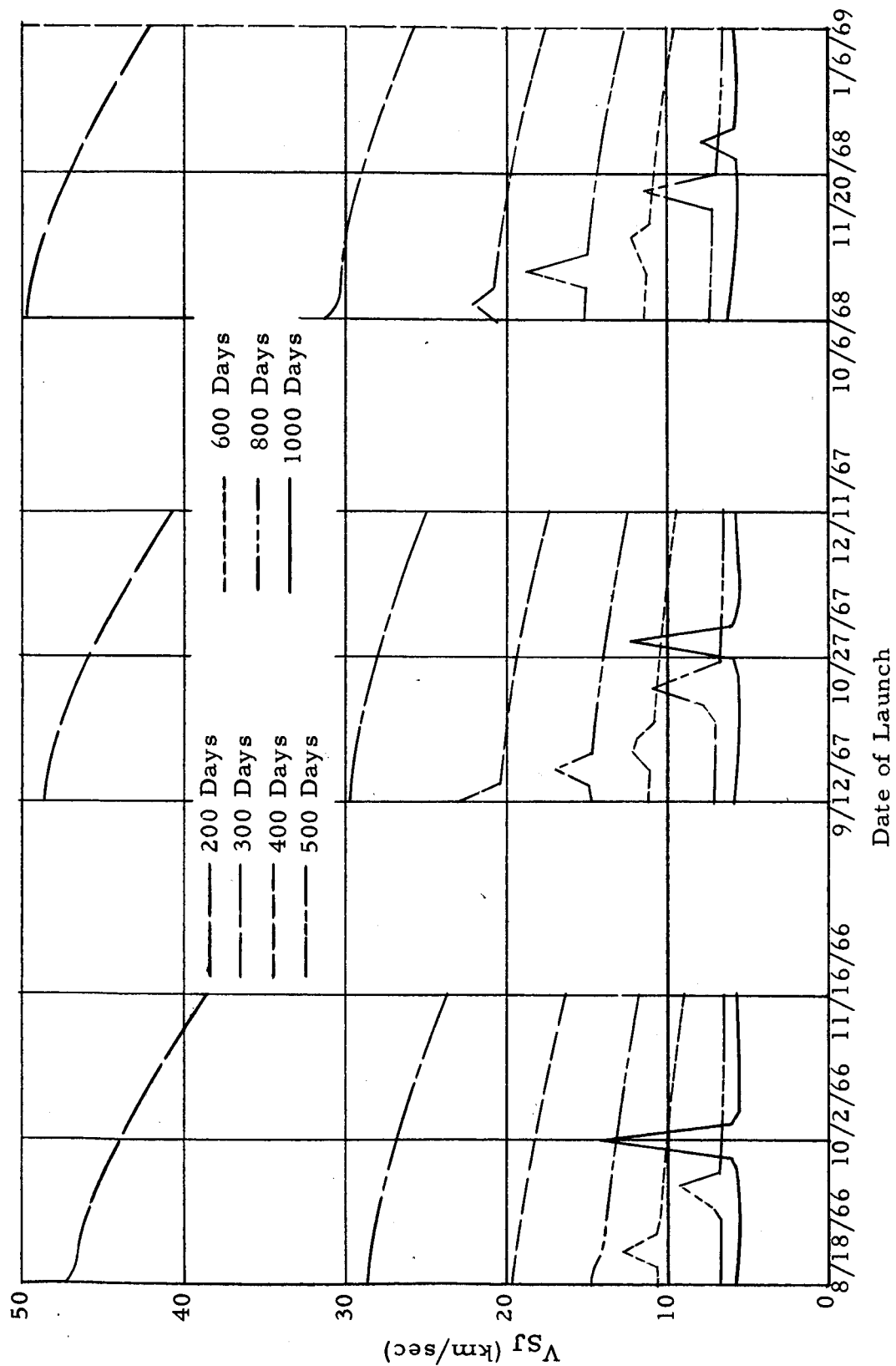
Fig. 22 V_{SJ}, SPACECRAFT VELOCITY OF APPROACH TO JUPITER FOR 200, 300, 400, 500, 600, 800, 1000 DAY TIMES OF FLIGHT, LAUNCHING IN 1966 TO 1969 LAUNCH WINDOWS
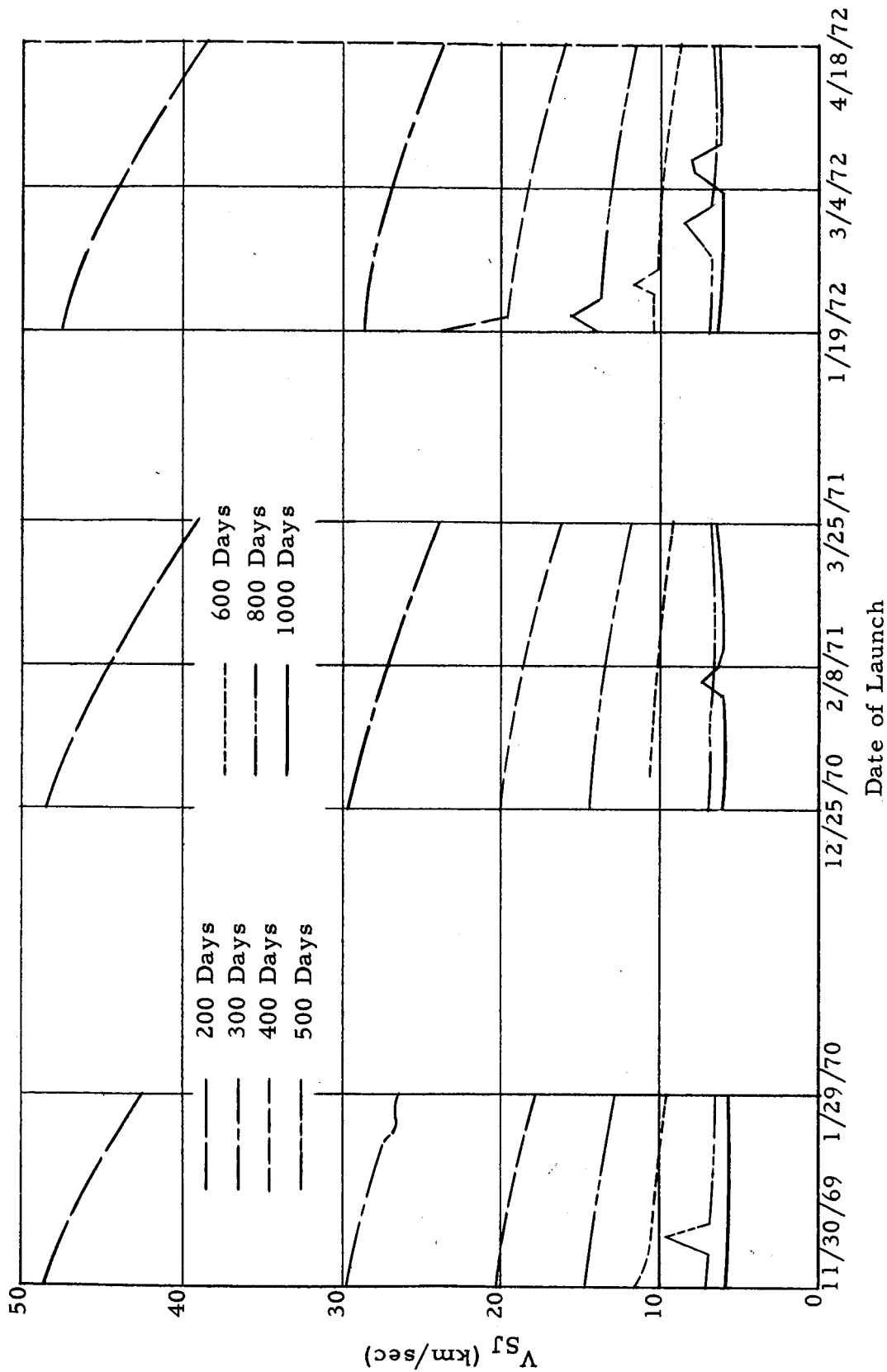
Fig. 23 V_{SJ}, SPACECRAFT VELOCITY OF APPROACH TO JUPITER FOR 200, 300, 400, 500, 600, 800, 1000 DAY TIMES OF FLIGHT, LAUNCHING IN 1970 TO 1972 LAUNCH WINDOWS
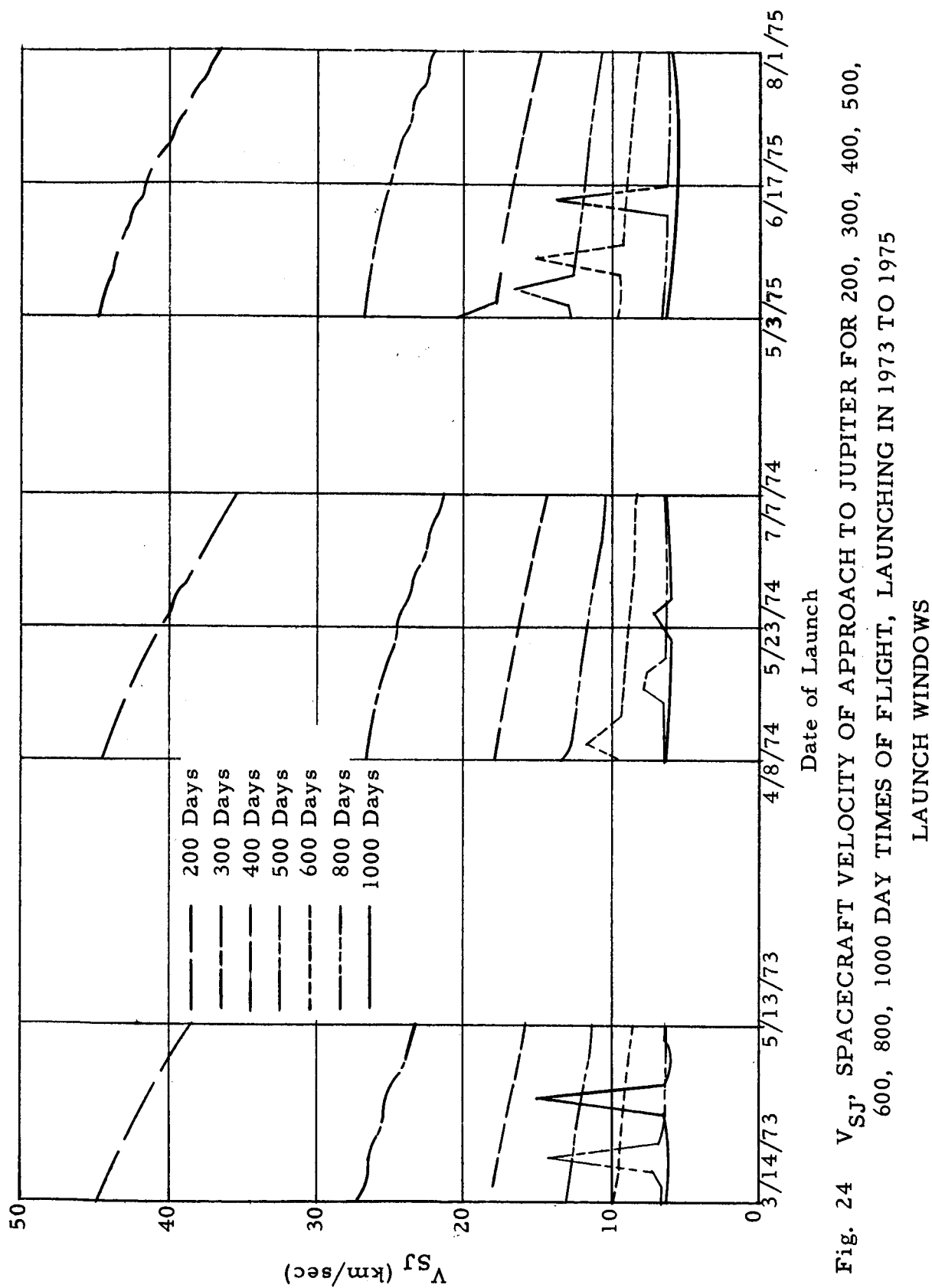
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Fig. 24 V$_{SJ}$, SPACECRAFT VELOCITY OF APPROACH TO JUPITER FOR 200, 300, 400, 500, 600, 800, 1000 DAY TIMES OF FLIGHT, LAUNCHING IN 1973 TO 1975 LAUNCH WINDOWS
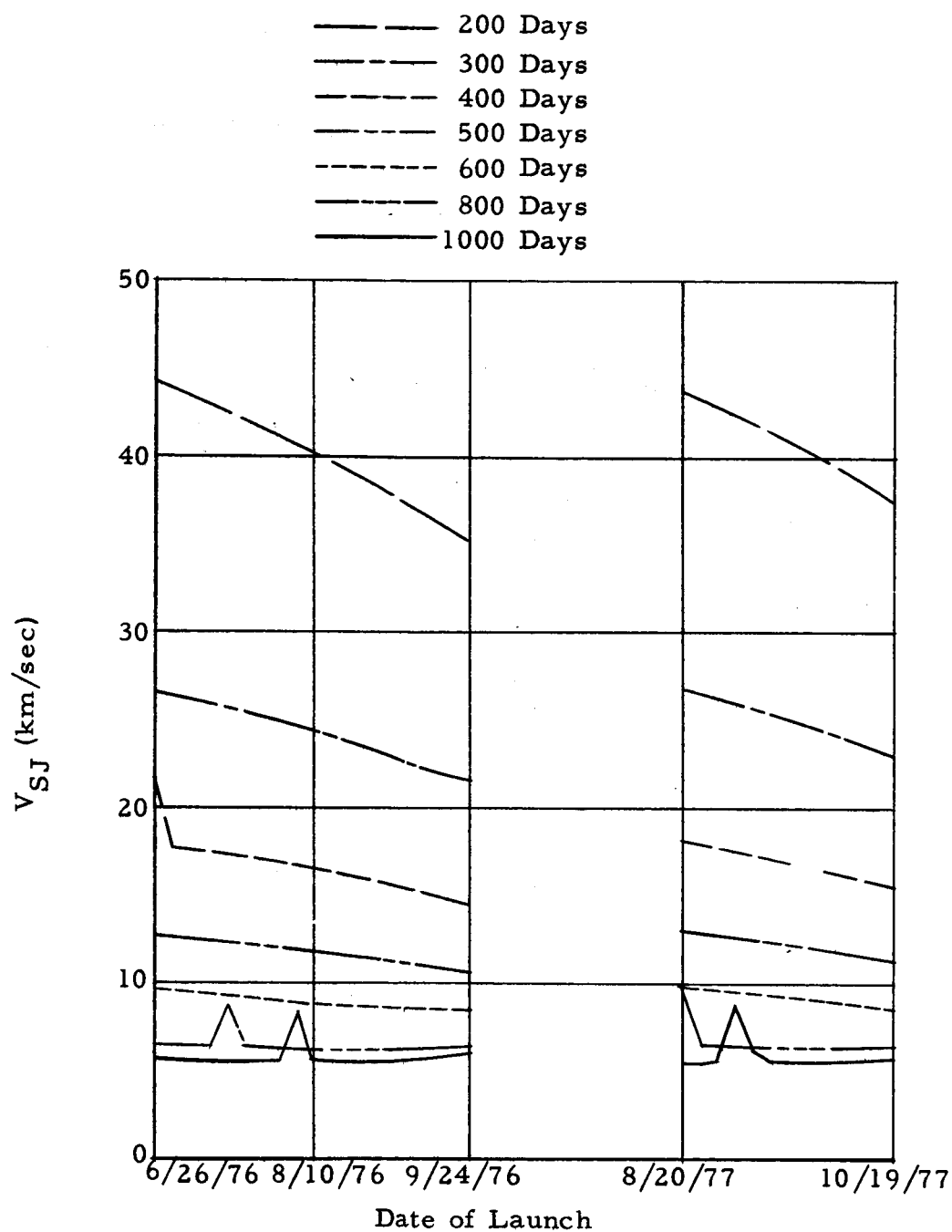
Fig. 25 V_{SJ}, SPACECRAFT VELOCITY OF APPROACH TO JUPITER
FOR 200, 300, 400, 500, 600, 800, 1000 DAY TIMES OF
FLIGHT, LAUNCHING IN 1976 AND 1977 LAUNCH
WINDOWS

R_p = perijove distance

B = asymptotic miss distance

R_a = apojove distance

Spacecraft elliptical with respect to Jupiter

Spacecraft hyperbolic with respect to Jupiter with velocity $V_\infty = V_{SJ}$

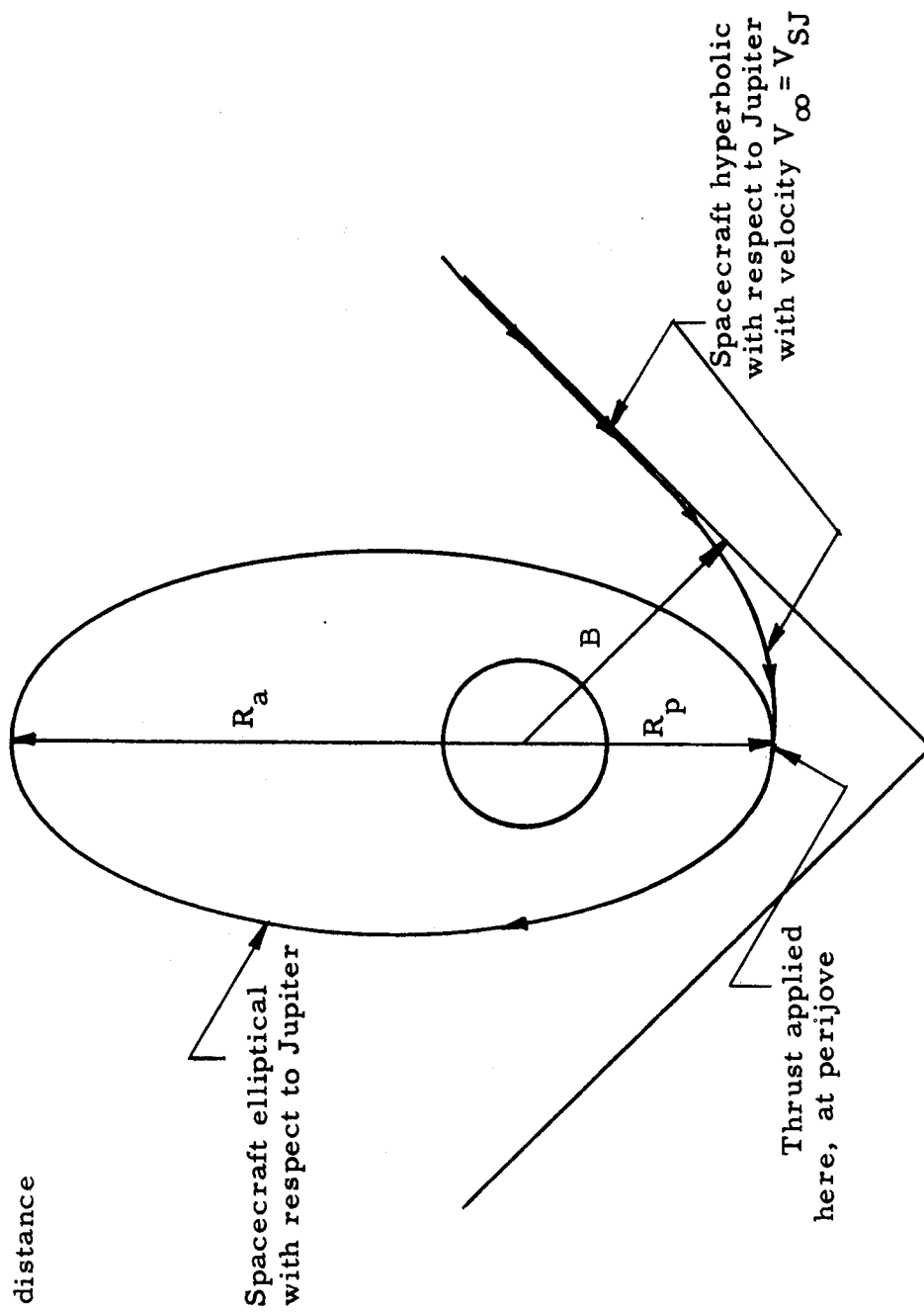Thrust applied here, at perijove

R_a

B

R_p
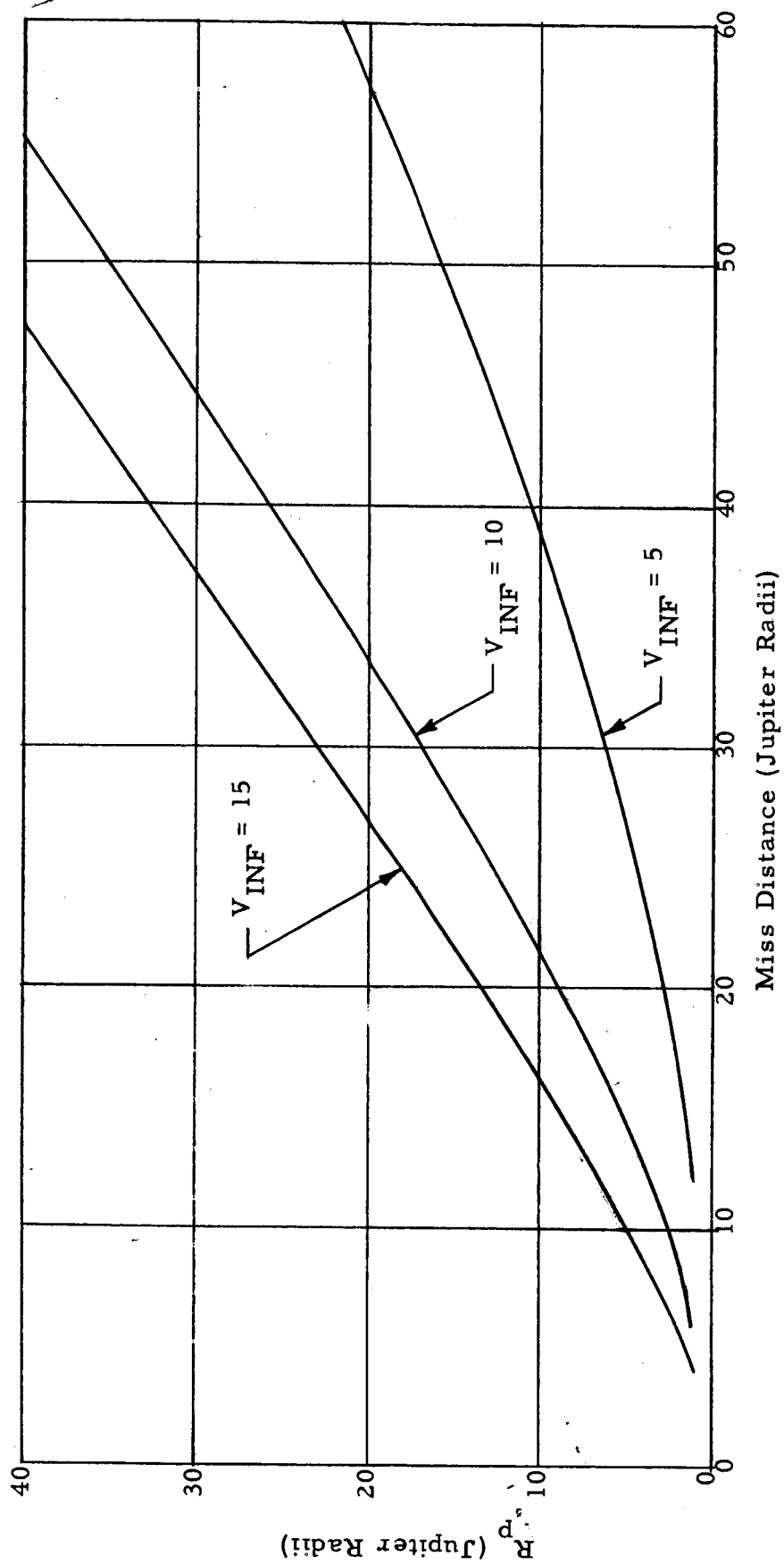
Fig. 26 NEAR JUPITER TRAJECTORY: DEFINITION OF TERMS

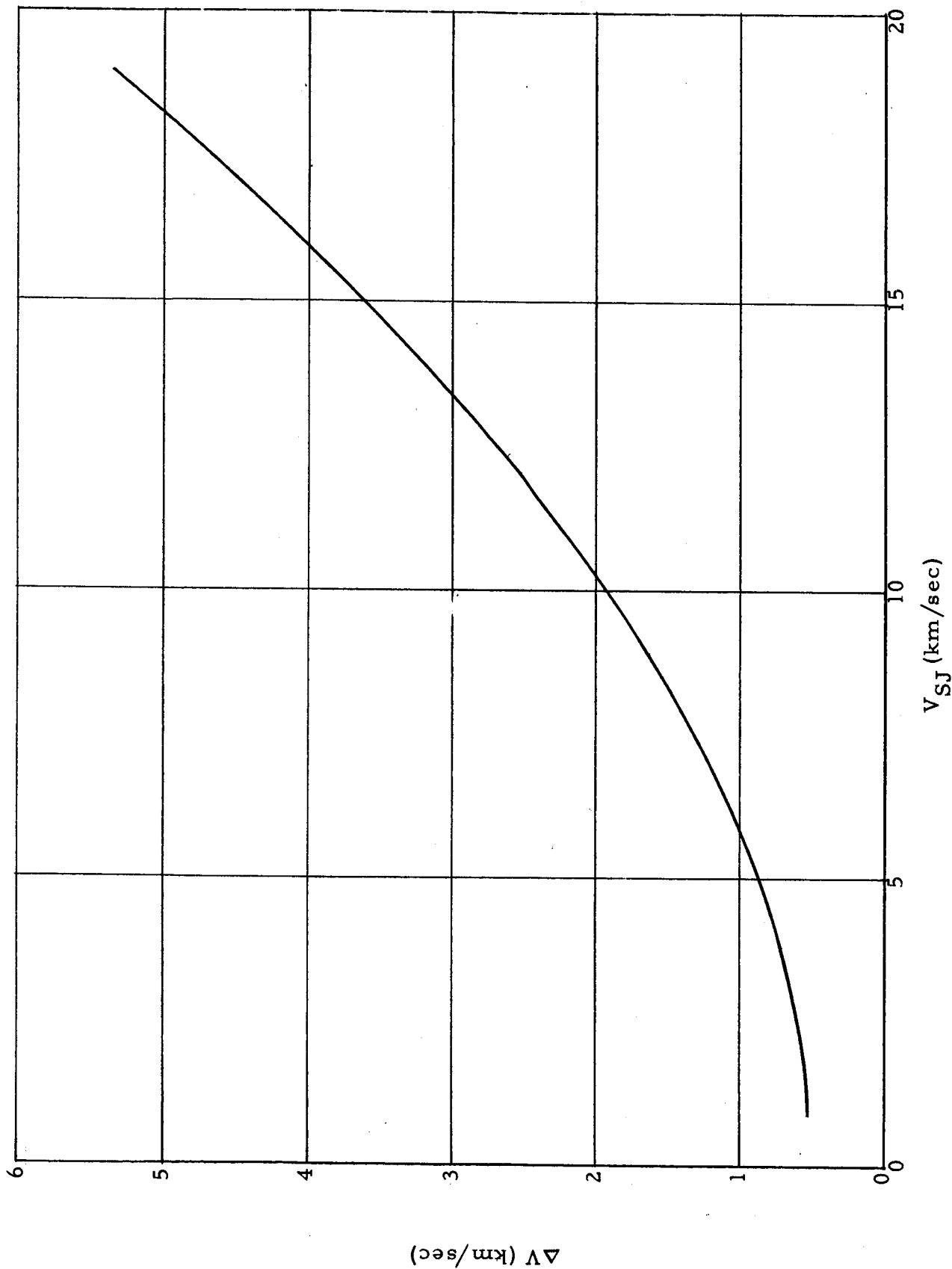Fig. 27   SPACECRAFT PERIJOVE VS SPACECRAFT ASYMPTOTIC MISS DISTANCE
FOR VARIOUS V_{SJ}

Fig. 28 ΔV, APPLIED AT PERIJOVE, NECESSARY TO CHANGE SPACECRAFT ORBIT FROM HYPERBOLIC TO ELLIPTICAL WITH RESPECT TO JUPITER VS V_{SJ}, FOR ELLIPTICAL ORBIT WITH $R_p = 3$ JUPITER RADII, $R_a = 100$ JUPITER RADII, PERIOD = 44 (EARTH) DAYS
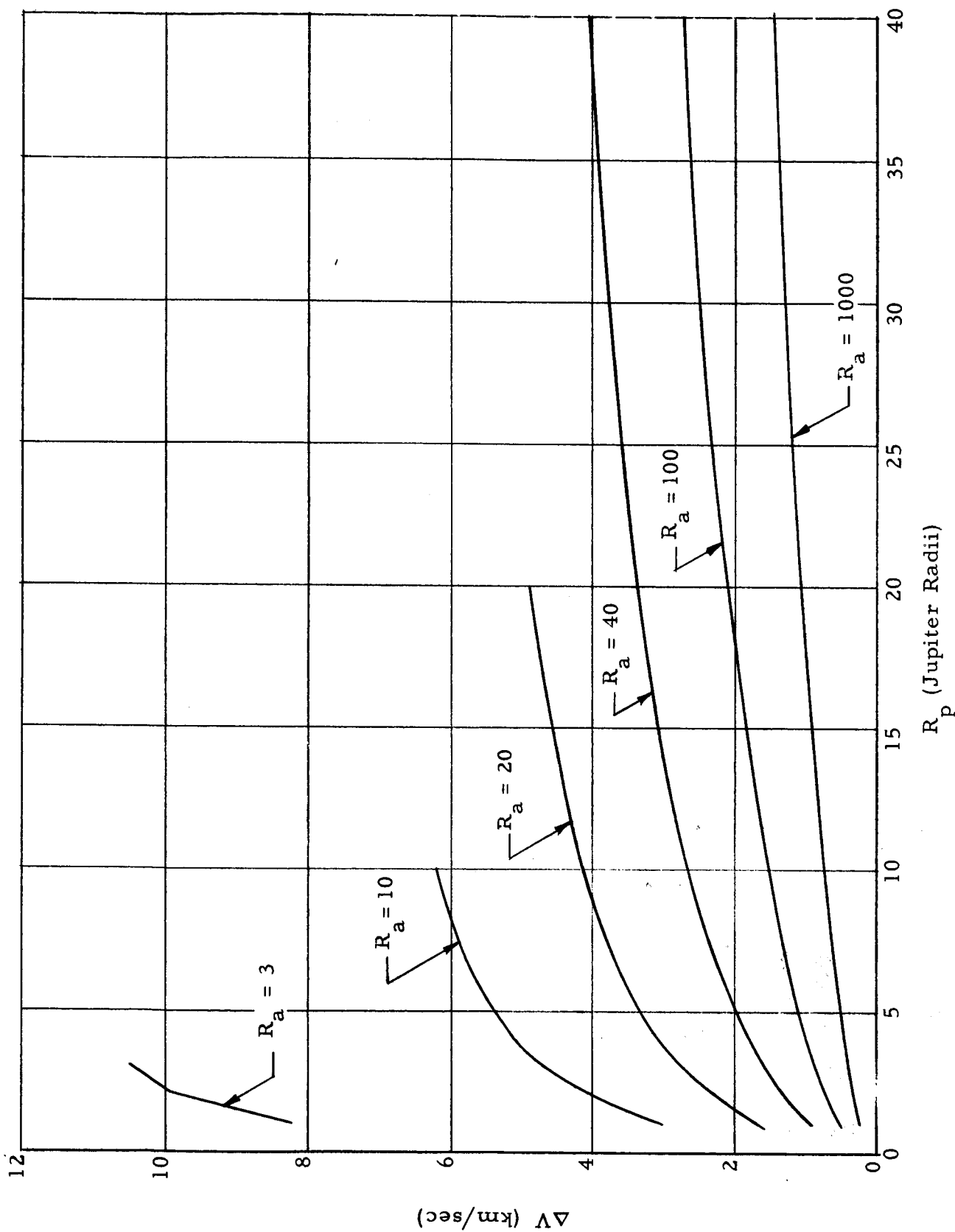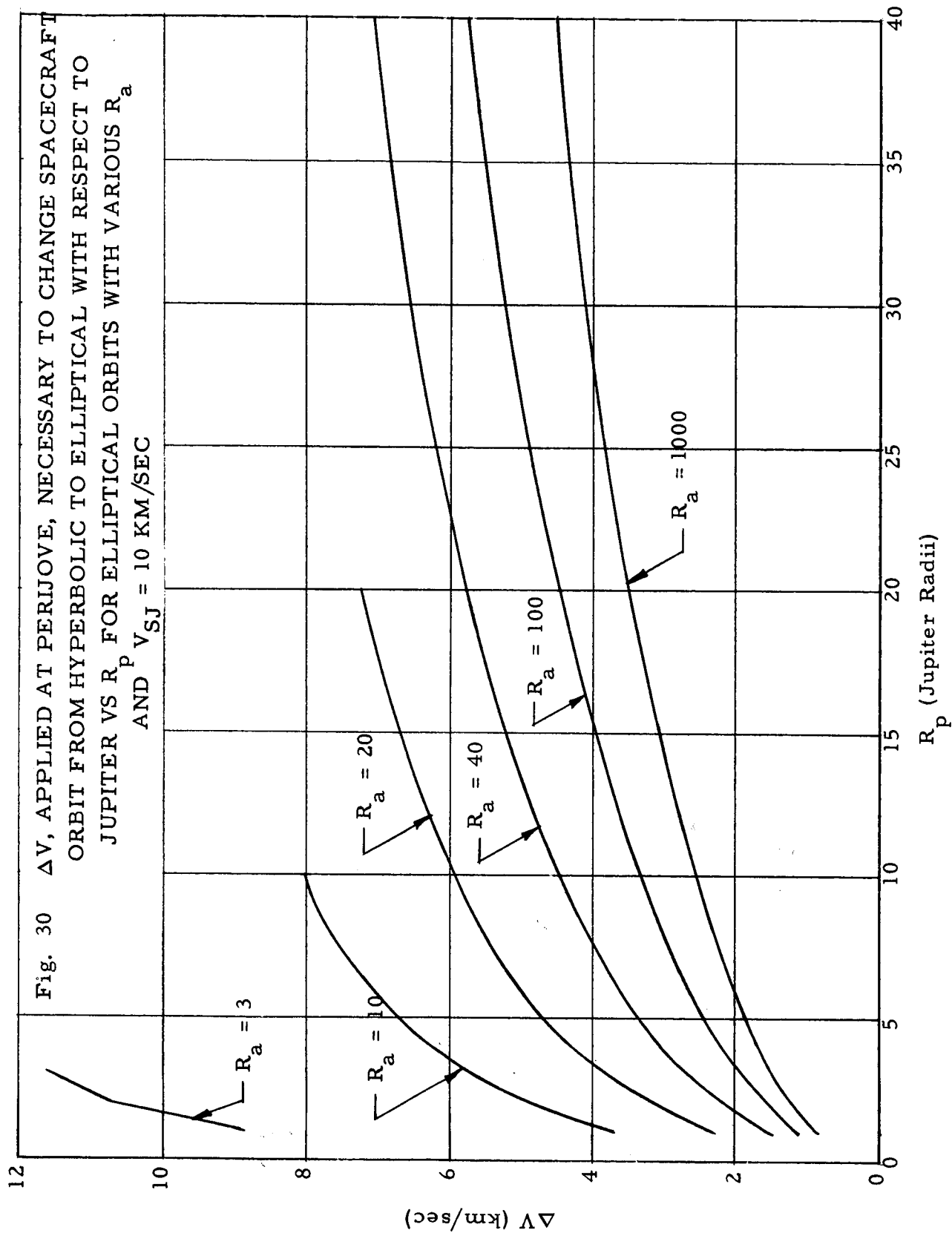
Fig. 29   ΔV APPLIED AT PERIJOVE, NECESSARY TO CHANGE SPACECRAFT ORBIT FROM
HYPERBOLIC TO ELLIPTICAL WITH RESPECT TO JUPITER VS R_p FOR ELLIPTICAL
ORBITS WITH VARIOUS R_a FOR $V_{SJ} = 5$ KM/SEC

Fig. 30 ΔV, APPLIED AT PERIJOVE, NECESSARY TO CHANGE SPACECRAFT ORBIT FROM HYPERBOLIC TO ELLIPTICAL WITH RESPECT TO JUPITER VS R_p FOR ELLIPTICAL ORBITS WITH VARIOUS R_a AND $^p V_{SJ} = 10$ KM/SEC

Fig. 31 ΔV, APPLIED AT PERIJOVE, NECESSARY TO CHANGE SPACECRAFT ORBIT FROM HYPERBOLIC TO ELLIPTICAL WITH RESPECT TO JUPITER VS R_p FOR ELLIPTICAL ORBITS WITH VARIOUS R_a AND V_{SJ} = 15 KM/SEC
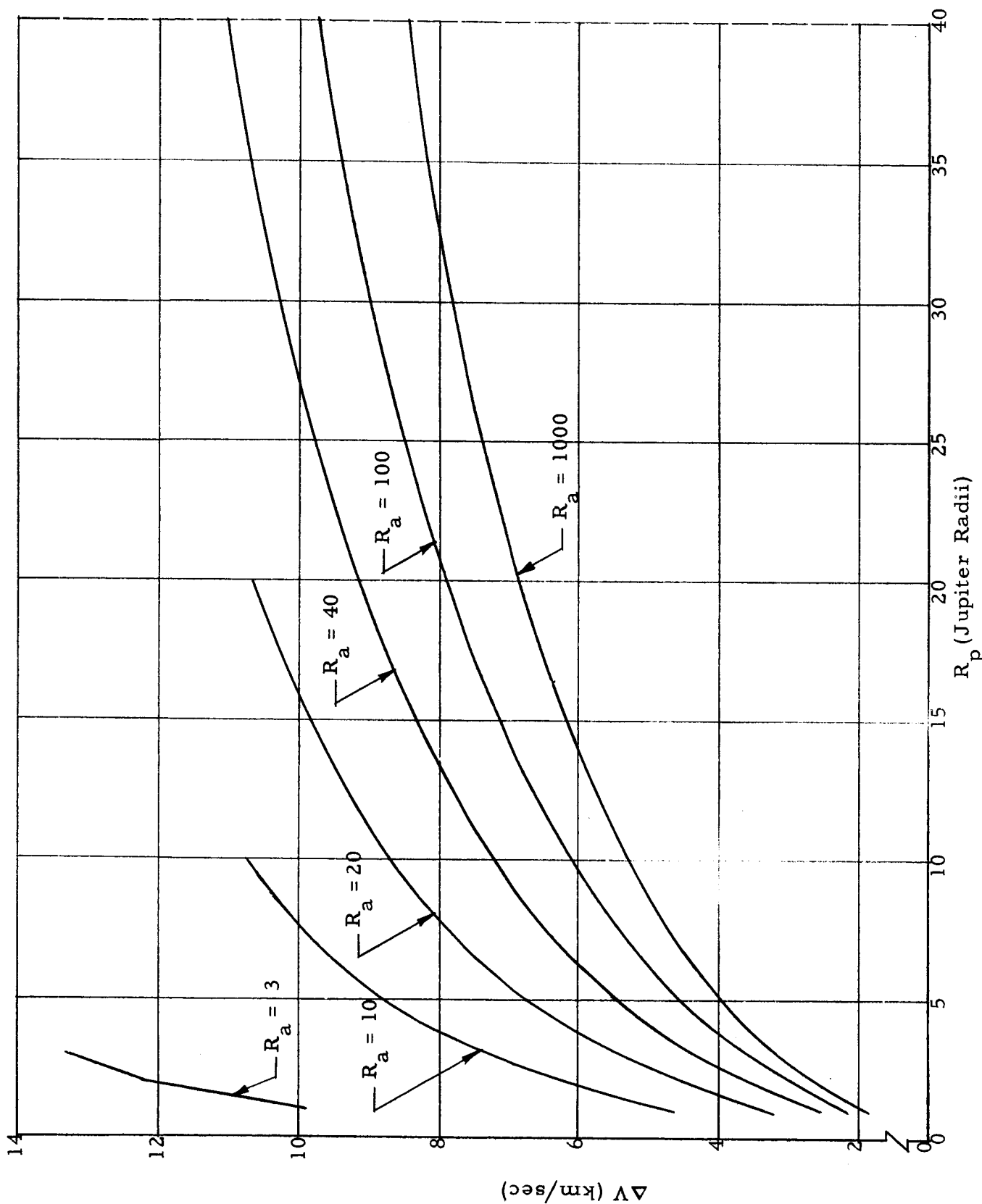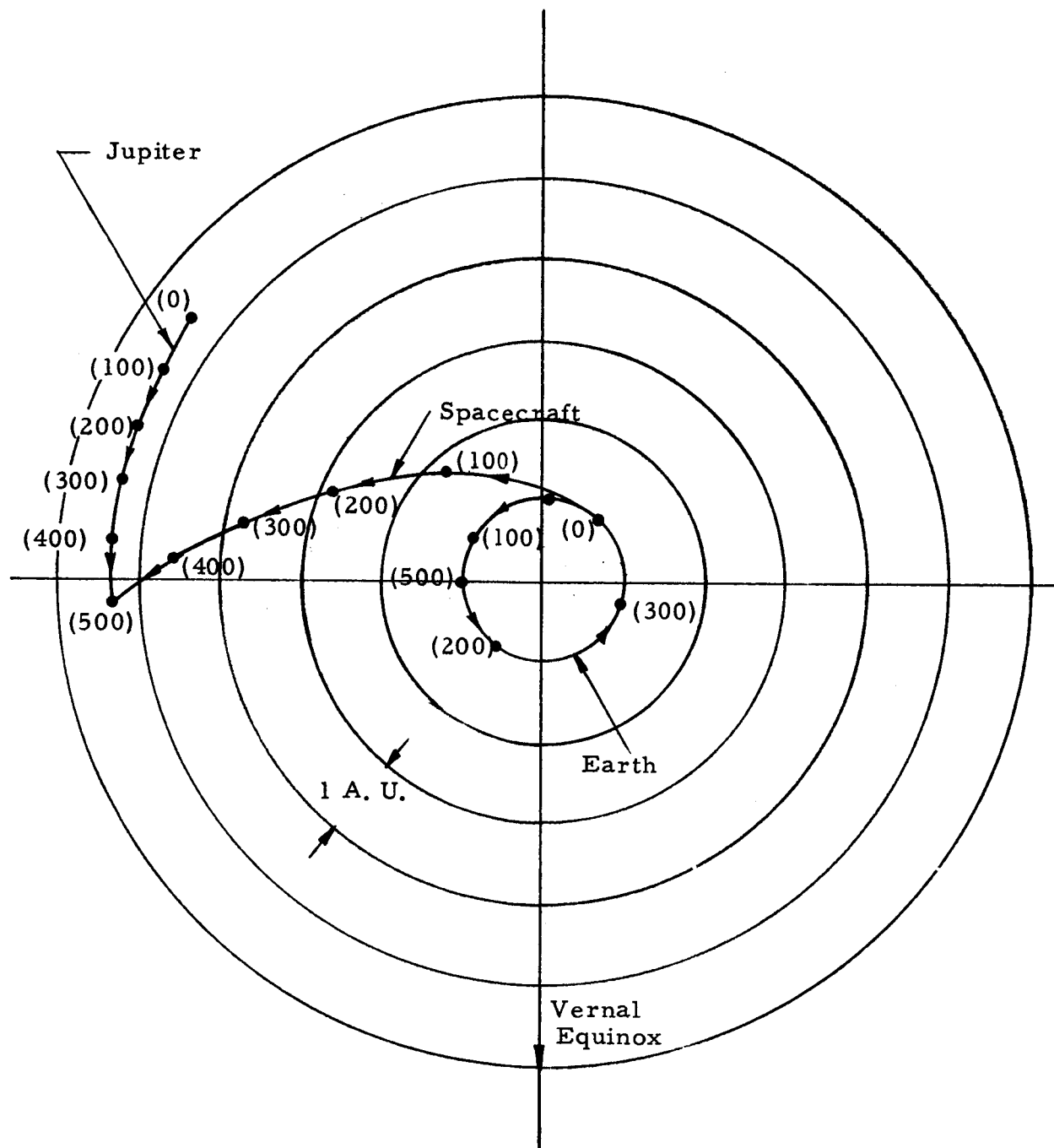
Fig. 32   EARTH, JUPITER, SPACECRAFT POSITIONS FOR TYPICAL
500 DAY FLIGHT, LAUNCH FEB. 8, 1971